Moral Hazard with Excess Returns

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Abstract

Consider a firm's manager who faces a moral hazard problem *and* can trade shares of the firm in the market. This special type of trader, here called the *distinguished player*, can influence the value of the firm subject to private effort costs. This article demonstrates how the different incentives between outside investors and the distinguished player imply that rational (and irrational noise-) traders systematically trade shares of such a firm at a discount below its equilibrium value if the distinguished player can trade anonymously. Buyers of this asset thereby earn *excess returns* on their investment. The resulting prediction that investment in companies with a distinguished player yields excess returns is consistent with empirical evidence in von Lilienfeld-Toal and Rünzi (2007) who report positive abnormal returns for owner-manager firms taken from S&P500 and S&P1500 firms.

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We call a *distinguished player* a manager of a firm who can trade shares of the firm on the stock market *and* can influence the value of this firm by exerting costly effort. Standard no-arbitrage equilibrium reasoning of asset pricing suggests that all relevant information should be priced in, including the ownership and the according optimal effort of the manager. This, however, leads to a paradox. If the privately costly effort is priced in, the manager would be better off to sell his shares at this price, not exerting effort, and saving the private effort costs instead. From here we call this paradox the *distinguished player paradox*. A related paradox is discussed in Grossman and Stiglitz (1980). In Grossman and Stiglitz (1980), information acquisition is costly (and does not have an impact on firm value) while in our model effort is costly and has an impact on firm value.[1]

The distinguished player paradox turns up in the empirical evidence in von Lilienfeld-Toal and Rünzi (2007) who show that standard arbitrage free asset pricing cannot explain the cross section of stock returns for firms with an owner-manager. For example, they show that a value-weighted portfolio consisting of all S&P 500 firms (1994-2005) in which the CEO holds more than 10% of the company's stocks significantly outperforms the total market portfolio by 13% p.a. after controlling for standard risk factors.

In this article we formalize the distinguished player paradox and propose a solution to the theoretical paradox that is also consistent with the empirical evidence. We follow the literature on asset pricing with large shareholders (see e.g. Bolton and von Thadden (1998), DeMarzo and Urosevic (2006), or Admati, Pfleiderer and Zechner (1994)) and analyze a market with a *distinguished player* who can trade on the stock market *before* directly influencing the firm's value by exerting costly effort. In this literature the distinguished player operates in a standard moral hazard context. For example, the distinguished player could be interpreted as the *agent* in the Grossman and Hart (1983) model. We now study the consequences of a distinguished player for the *equilibrium trade price* of the firm before the moral hazard problem is resolved.[2]

Our most important departure from the previous literature is here to remove the assumption that *firm value* and *trade price* must coincide. Rather, value and price are both determined endogenously. Firm value depends on the effort decision of the

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[1] Krebs (2007) and Muendler (2007) have shown that the the Grossmann-Stiglitz-paradox disappears — i.e. price can be fully revealing with costly information acquisition — if finitely many players can use mixed strategies. We consider both, a setting with finite number of players and a setting with an infinite number of players and show that the distinguished player paradox is more difficult to overcome in the sense that neither finite models nor mixed strategies can resolve the incentive asymmetry between distinguished player and outside investors. The most important reason for this difference is that it is present even with complete and symmetric information.

[2] As the previous literature we are mostly interested in the pricing predictions of a market before the moral hazard problem has been resolved since otherwise the distinguished player paradox disappears. The latter case obviously corresponds to the standard case without a distinguished player which is a special case in our formulation. In this sense our results can be interpreted as a robustness check of standard asset pricing methodology with respect to the introduction of agency problems.

distinguished player which in turn depends on his ex post ownership determined by the outcome of the market. In contrast, *equilibrium trade price* is determined by the market mechanism clearing aggregated demand and supply. In our first setting with a finite set of traders we allow for strategic interaction of rational traders in a general market game and derive a pricing prediction. In our second setting we study the robustness of our pricing prediction in a continuum trader framework with noise traders where outside investors lack strategic impact on the trade price.

To discriminate the possible pricing predictions of the distinguished player paradox two classes of trade equilibria are of particular interest, *true value* and *excess returns equilibria*. In a true value equilibrium, shares of the firm are traded at the price that equals the equilibrium value of the firm. In excess returns equilibria, shares of the firm are traded at a price strictly below the equilibrium value of the firm.

Our main results. (1) Trading shares of a firm at a price equal to the equilibrium value is not an equilibrium in rational call auction markets — i.e. in our language there is no *true value trade equilibrium*. This formalizes the aforementioned paradox of endogenous firm value and arbitrage free asset pricing. However, (2) there exist *excess returns equilibria* where traders buy and sell at a price strictly below the equilibrium value. This provides an explanation for the abnormal returns of owner-CEO firms as reported in von Lilienfeld-Toal and Rünzi (2007). Further, (3) excess returns equilibria are robust with respect to introducing (i) trading costs, (ii) noise traders and price taking behavior, and (iii) the specification of the market microstructure. (4) Conversely, excess returns equilibria do not exist without a distinguished player. (5) Together this yields what we call the *distinguished player hypothesis*: Investments in firms with a distinguished player systematically outperform those in firms without a distinguished player and thereby the whole market. The main ideas behind these results are now motivated in more detail.

Why is trade at the true value not an equilibrium? Suppose to the contrary that shares of the firm are traded at the correct value from the outside investor's perspective. Then, the distinguished player wishes to sell his shares due to his lower valuation caused by private effort costs while outside investors are indifferent between trading and not trading. In an anonymous market this cannot be an equilibrium. The distinguished player decreases effort and saves effort costs if he manages to sell some shares. This implies that the distinguished player strictly gains if he can sell without having a significant impact on the share price. We show that *anonymity* in a market — i.e. the fact that traders do not observe the identities of other traders — guarantees that the distinguished player indeed can sell shares without affecting the price. Hence, trade at a price equal to equilibrium value is not an equilibrium in this set-up and a distinguished player's effort decision systematically cannot be "priced in" correctly. These incentives are relevant since trade is anonymous in real world stock exchanges.

Excess returns equilibria. Since this is a negative result the obvious question is: Are there other equilibria? Yes. Albeit their structure is less obvious we show that they generally share several common features. Their most salient property, however, is that the equilibrium trade price is *strictly below* the equilibrium value and therefore equilibrium-buyers strictly gain by buying an object below its value. Therefore we call these equilibria *excess returns equilibria.*

The observation that excess returns equilibria are the only consistent outcome of a market with a moral hazard problem has several novel implications. Most obviously, in contrast to standard asset pricing theory "no-arbitrage" here is not synonymous with the notion that the market price equals the equilibrium value. Since rational equilibrium-buyers strictly gain even without any informational advantage excess returns are inconsistent with the standard definition of *efficient markets* and *no-arbitrage* in equilibrium (see for example Fama, 1970 or Ross, 1976). The immediate question is here, why does not everybody buy maximally at a price for which buying yields strictly positive gains? A closer game theoretic inspection reveals that in an excess returns *equilibrium* "no-arbitrage" is still valid in the more general sense that no rational investor can gain by buying or selling more or less although being aware of the fact that shares are traded below their value. The reason for this is that rational investors anticipate that the distinguished player has an incentive to sell shares — or buy less — once the share price exceeds a certain threshold and that this threshold is below the equilibrium value. Hence, trade at the equilibrium value would encourage the distinguished player to sell his shares in an anonymous market and save on effort costs instead. We show in this article that this logic prevails in two different settings, (i) a fully rational, strategic and (ii) in a noisy, price taking setup. The two settings provide two different explanations for the same prediction.

In the fully rational environment (i) excess returns equilibria are characterized by the property that any deviation that drives up the market price towards the equilibrium value triggers the distinguished player to sell and decrease effort instead of raising the company value to the anticipated level which in turn causes even bigger losses to everybody. This latter property of excess returns equilibria is called *pivotalness.* Any failure to coordinate on a sufficiently low market price below the true value destroys wealth for all shareholders by removing incentives for the distinguished player to exert effort and generate positive externalities. This explanation requires a high degree of traders' rationality and particularly their awareness of their strategic influence. One question then is whether this high degree of rationality is necessary to support this explanation.

Surprisingly this is not the case as our setup (ii) demonstrates. We show that the downward pressure on equilibrium prices caused by the economic incentives for a *distinguished player* neither rests on *full rationality* of all traders and with it *pivotalness* nor on the *strategic influence of traders* on prices. We demonstrate this in a continuum-trader-version of the model with noise traders. Within the so defined stochastic environment we can again show existence of excess returns equilibria for several reasonable market mechanisms. The basic idea prevails but with the following twist. With noise final allocations and prices are random. As before, the distinguished player plans to sell shares whenever the share price exceeds a certain threshold which now occurs with positive probability. As before, rational outside investors do not wish to buy shares at prices above this threshold because they anticipate that the distinguished player reduces costly effort. As a result, small rational outside investors cannot gain by bidding up the share price even though it is feasible. Consequently, shares are underpriced in expectation. Note that this latter logic does not rest on pivotalness. In contrast to the strategic setting for small price takers now it is not rational to sell below the true value. Therefore, irrational noise traders are necessary to generate trade and liquidity. Hence, even in a price-taking environment with noise the distinguished player's incentives to sell his shares impose downward pressure on the trade price. This provides a second theoretical explanation for excess returns equilibria.

Market microstructure. Our framework establishes progress in another direction. It is well known that most results in the market microstructure literature hinge critically on the specification of the market mechanism. For example, O'Hara (1995) discusses different market microstructure models in the literature and states (p. ix) that the "generality of their results, and hence their applicability, is not well understood". This lack of understanding is indeed bothering since any specific market mechanism is only an approximation of real world trading systems and it is not known what constitutes a good approximation. Without more general results we cannot judge which theoretical conclusions depend on the market mechanism and which don't. Moreover, if they depend on it, how? While our existence results — as the previous literature — depend on the specific market mechanism our characterization results in section 4 show that some of the surprising predictions on asset pricing with a distinguished player are valid generally, i.e. for any market mechanism. Moreover, we formulate our results regarding the pricing of stocks for companies with a distinguished player in section 3 for pricing and allocation rules that are used in the real world. The differences in the corresponding existence proofs clarify to which extent the details of the relevant equilibria beyond our characterization results in fact do depend on the market mechanism. Any existence or non-existence result must specify the strategy spaces in full detail. Since this theory derives non-standard pricing predictions about stock markets we wanted to understand better the implications of the differences in the allocation rules of various real world exchanges. Realism with regard to allocation rules, however, comes with the price of adding further technicalities. Our technical appendices provide a wealth of details for those readers interested in them.

Anonymity and institutional or contractual clauses. Anonymity of the market plays a salient role for our reasoning. To which degree is anonymity realistic? It is difficult to argue that in anonymous markets distinguished players differ from other traders in their ability to commit not to trade. In reality neither SEC regulations nor privately stipulated contracts can fully rule out anonymous trading by distinguished players. In fact, the majority of shares held by executives are common shares that are not subject to a non-selling clause. Furthermore, SEC regulations force managers to report trade ex post or sometimes requires insiders to announce plans to trade ex ante. Even then, these shares are then traded anonymously on the market. In particular, for our reasoning it is crucial that outside investors do not know whether they buy from insiders or from other outsiders. Due to this theory already some outside investors' knowledge about the incentives of a distinguished player are sufficient to support excess returns equilibria. Given the wide applicability and acceptance of the *distinguished-player-concept* in corporate finance we believe that it is important to better understand its relevance for asset pricing.

The paper proceeds as follows. Section 1 introduces the main idea by an example. Section 2 establishes the reference model, introduces formally the idea of a distinguished player, and sets up the notation for general market mechanisms. Any particular market mechanism specifies a corresponding market game. In section 3 we show that trade at the true value does not occur in equilibrium. However, excess returns equilibria exist for several real world call auctions. In section 4 we characterize excess returns equilibria for general market mechanisms. Section 5 derives robustness results and extends the existence of an excess returns equilibrium to a world with a continuum of traders and noise traders. Section 6 discusses in more detail the relationship of this theory to the related empirical and theoretical literature. Section 7 discusses some extensions like multiple distinguished players or a market maker while section 8 sums up. Appendix A explains more rigorously the rich structure of strategies in the market game, Appendix B extends the language to stochastic market mechanisms while Appendix C contains all remaining proofs.

1 Intuitive Example

Consider three rational players $i \in I = \{0, 1, 2\}$ who jointly own a business project. Suppose for simplicity that initial ownership of the project consists of three indivisible shares of equal size $(\alpha_0, \alpha_1, \alpha_2) = (1, 1, 1)$. Imagine that players $i = 1, 2$ are wealthy investors in contrast to player $i = 0$. Player $i = 0$ — called the *distinguished player* — has a brilliant idea how to raise the value of the project from $\underline{v} = 0$ to $\bar{v} = 30$. To implement and materialize this idea the distinguished player has to work hard and exert effort $e \in \{0, 1\}$ facing private effort cost $c(e) = 4e$. Finally, the project is sold for its terminal value, i.e. each share is worth $10e$. Everything is public information. Without trade this world is quite trivial, the distinguished player certainly exerts high effort $e = 1$ being aware of the fact that the final value of his share exceeds his private effort cost. Hence, the final value of each share is 10 and payoffs are $(u_0, u_1, u_2) = (6, 10, 10)$.

However, this was just the background story. The main object of interest in this article is an anonymous market for stakes of the project before the distinguished player decides on effort. The role of players $i = 1, 2$ in this example is to perform a very simple version of this anonymous market. Instead of friends or business partners we now imagine nameless anonymous shareholders.

Market game. While the rules of the market game in this example are specific and simple they already display some properties of real world stock markets as anonymity, trade volume maximization and price priority. Every player i simultaneously can either do nothing or announce one limit order. This order can either be a buy order of quantity 1 using a limit price $p_i^b \in \{0, 1, \ldots, 10\}$ or a selling order of quantity 1 using the limit price $p_i^s \in \{0, 1, \ldots, 10\}$. The distinguished player in this example cannot afford to buy. Therefore, $i = 0$ can only submit a sell order or do nothing. The market is assumed to clear as follows. There is trade if and only if at least one buying order p_i^b and one selling order p_j^s are submitted such that $p_j^s \le p_i^b$. If there are more than one competing buy orders with different limit prices and one sell order, such that all buy prices are at least as high as the sell price only the buy order with the higher price is executed against the sell order. If the buy order prices coincide each of them is executed with equal probability. Correspondingly, if there are one buy order and more than one sell order with limit prices below the buying price only the lower sell order is executed, or again, if identical all are executed with equal probability. Finally, if there is more than one price maximizing the trade volume the market mechanism picks the lowest such price.

Is trading at the true value an equilibrium? We first show that to trade at the high "true" value $p^* = 10$ cannot be an equilibrium. If there were such an equilibrium there must be a buy order of a non-distinguished player $i = 1, 2$ with $p_i^b = 10$ and at least one sell order $p_j^s = 10$ of another player $j \neq i$. Note first that player j cannot be the distinguished player since otherwise i could improve by not submitting a buy order. Hence, j is the other outside investor. However, this cannot be an equilibrium either since in this case the distinguished player can gain by submitting a sell order at $p_0^s = 9$. The price priority rule of the market mechanism makes sure that this order is executed and the distinguished player exerts low effort in turn. This, however, yields a market price $p^* = 9$ strictly below 10 which contradicts $p^* = 10$. The less interesting *true value* case where players trade at $p^* = 0$ and the distinguished player exerts low effort cannot be an equilibrium either for a similar argument. One might imagine that some sort of mixed strategies could help out to construct a true value equilibrium. It is obvious that mixing in effort does not help since after any realization of trade any mixed effort decision is strictly dominated by a pure effort decision, i.e. no effort if the distinguished player managed to sell and full effort if he did not sell. It is a little more tricky to see that mixing in the trading game cannot yield a true value equilibrium either. The idea of the proof (of our much more general theorem) is first to recognize that if all outside investors play pure strategies the distinguished player's ex-post ownership is deterministic. Therefore, to obtain stochastic ex-post ownership for the distinguished player the distinguished player himself and at least one outside investor must be simultaneously indifferent between several pure trading strategies. However, this cannot be the case since their private valuations of the traded object differ once the distinguished player exerts effort with positive probability.

Excess returns equilibrium. Is there any other trade equilibrium where players do trade at a price that does not reflect the equilibrium value of the traded object? The answer is: Yes, for example at $p^* = 6$. To understand this equilibrium consider a situation in which the distinguished player submits a sell order at price $p_0^s = 7$. This looks like a decent strategy since it can only raise his payoff compared to the payoff of not trading which is 6. Now, suppose that player $i = 1$ submits a buy order $p_1^b = 7$. Although this behavior superficially appears to be risky since the distinguished player can sell at this price it turns out to be quite smart. The reason is that the unique best response of player $i = 2$ is now to submit a sell order $p_2^s = 6$. According to the market mechanism this implies market price 6 and yields player $i = 2$ a payoff of 6 and player $i = 1$ a payoff of 14.

To see that selling is indeed optimal for player $i = 2$ consider any deviation. A deviation will either result in a lower price which makes player $i = 2$ worse off. Alternatively, player $i = 2$ could submit a higher price (or no sell order at all). Then, whenever player 2 does not sell his shares, the distinguished player will sell instead. In this event the distinguished player will exert low effort and the pay-off of player $i = 2$ would be 0. This makes player $i = 2$ worse off.

Next, the distinguished player cannot improve because selling at a price of 6 yields the same payoff as not trading at all. Finally, player $i = 1$ cannot benefit from changing his order either. He cannot buy at a lower price and not buying would be worse.

It is easy to check that there are further equilibria. Clearly, the roles of the *equilibrium winner* $i = 1$ and *equilibrium looser* $i = 2$, i.e. the players who realize strict gains and losses relative to the equilibrium value by their trading behavior, may be permuted. There are also other equilibrium prices, all of them strictly below equilibrium value. However, to develop the full structure of this example does not yield much more additional insight for the general setting. More interesting is to add another player who behaves "irrationally" and trades for some exogenous reason, for example a liquidity shock. It is not difficult to see that this imposes trade with strictly positive probability, and that the same observations regarding expected prices and values carry over to such an extension with noise. However, to keep this example short we sum up our main insights and postpone this latter route to the general framework in section 5.

To wrap up, in this setup everybody knows that high effort of the distinguished player is efficient. If the project is traded at a price equal to the according high true value the distinguished player would rather prefer to sell at this or even a lower price and then exert low effort. Therefore, trade at the high *true value* is not an equilibrium. More generally, if there is trade at the price that reflects the true value of the object, i.e. the value from the perspective of an outside investor there exists always a trader who can strictly improve, either the distinguished player or an outside investor. It is our first main result that non-existence of a true-value equilibrium is indeed a very robust observation. However, there exist *excess returns equilibria* where a buyer enjoys strict excess returns on his investment. More generally, the incentives of a distinguished player with the associated payoff externalities are inconsistent with some crucial concepts of capital market theory, the efficient markets hypothesis and traditional no-arbitrage. Available and even *public* information about a project's value cannot be reflected by the trade price in an anonymous market.

The remainder of this article shows that all the crucial observations in this example are surprisingly robust and are valid in much more general and realistic settings.

2 Market Game with a Distinguished Player

Distinguished player. Denote by $i = 0$ a *distinguished player* being interpreted as a "manager-owner", raider, activist shareholder, or founder of a firm. Further, denote by $i = 1, ..., N$ *outside investors* with (weakly) positive stakes in this firm.[3] *Distinguishedness* of a player-investor is defined as the ability to influence the value of the firm.[4] If the distinguished player exerts effort $e \in \mathbb{R}_+$ the firm's value is $v(e) = \underline{v} + e(\bar{v} - \underline{v})$. Exerting effort causes private effort costs $c(e)$ with $c' \geq 0$ and $c'' > 0$. To compare with standard asset pricing models, we allow for the case $\Delta v \equiv \bar{v} - \underline{v} = 0$ where no player is a distinguished player.

Firm ownership. Our object of investigation is a market game where stakes of the firm can be traded before the distinguished player decides on effort. The *initial ownership structure* of the firm is exogenously given. It is denoted by $\alpha = (\alpha_0, ..., \alpha_N)$ with $\sum_{i=0}^{N} \alpha_i = 1$. We suppose a discrete number M of indivisible shares. Hence, initially player i owns $\alpha_i M$ shares of the firm. The market game to be described subsequently endogenously results in the final ownership denoted by $\omega = (\omega_0, \omega_1, ..., \omega_N)$.

Effort choice. In the market game yet to be defined, stakes of the firm are traded before the distinguished player decides about his effort. Once the market game is over

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[3] To study the role of small investors and price taking behavior we study also a continuum of investors in section 5.

[4] For this theory it is irrelevant whether this influence is positive or negative or can go in both directions. Once it is negative one might want to choose different vocabulary. For example *exerting high effort* could be replaced by the notion *not stealing* to obtain the same incentive structure.

the distinguished player chooses effort to maximize the net value $\omega_0 \left(\underline{v} + e\Delta v \right) - c(e)$ of his final stake ω_0 in the firm. Let

$$e(\omega) = \underset{e}{\operatorname{argmax}}\, \omega_0 \left(\underline{v} + e\Delta v \right) - c(e)$$

denote the unique optimal ex-post effort choice of the distinguished player.

Similarly, the payoff of any outside investor $i = 1, \ldots, N$ after the market game is given as $\omega_i \left(\underline{v} + e\Delta v \right)$, i.e. the final value of his stake after the distinguished player's effort choice.

We follow the related literature, e.g. Admati, Pfleiderer and Zechner (1994) or DeMarzo and Urosevic (2006), and assume effort to be non-contractible. This approach is also supported by empirical evidence.[5] We believe that as long as the market price enters the distinguished player's incentives and the market provides some anonymity, the problems we discuss here cannot be perfectly alleviated by appropriately chosen contractual clauses. We therefore simply assume that no contractual provisions are used at all.

Prices and strategies. Suppose feasible prices are discrete and denoted as $p \in P := \{-\infty, ..., \underline{v}, \underline{v} + \delta, \underline{v} + 2\delta, ..., \bar{v}, ..., \infty\}$ with some exogenous tick size δ. Real world market mechanisms distinguish between buy and sell prices p_b, p_s. The difference $\gamma := p_b - p_s \geq 0$ is called *bid ask spread*.[6] The bid ask spread γ is supposed to be a non negative integer multiple of the tick size, i.e. $\frac{\gamma}{\delta} \in \mathbb{N}$. From here $p \equiv p_s$ always denotes sell prices whereas the corresponding buy price is $p_b = p + \gamma$. All buy orders depend on p_b and all sell orders depend on p_s.

Strategies or *market actions* $a_i \in A_i$ of an investor i are collections of buy and sell orders. Mathematically, a strategy a_i can be described by a pair $\{D_i(p), S_i(p)\}$ of set-valued demand and supply functions or correspondences. For example, $q \in S_i(p)$ represents a quantity of shares trader i is willing to sell at price p. Together, $Z_i(p) = D_i(p) - S_i(p) \subset Q$ is a set of positive or negative net quantities composed by demand and supply quantities that would be acceptable for investor i at price p.[7] We are particularly interested in orders consisting of market orders, limit orders and fill-or-kill orders. In appendix A we describe in more detail how the rich mathematical object of an excess demand correspon-

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[5] According to the execucomp database, there are 5106 officer year observations, where an officer owns more than 5% of unrestricted shares within all S&P 500 or S&P 1500 firms between 1992-2004. In contrast, only in 26 officer years an officer holds more than 5% of restricted shares. For a description of the data, see von Lilienfeld-Toal and Rünzi (2007).

[6] We explicitly want to include the case of bid ask spread $\gamma = 0$ to be able to compare this model with models without trading costs.

[7] To allow traders as in reality to choose demand and supply rather than just the sum of both – i.e. excess demand – opens the possibility for "beller strategies" in which a trader might, for example, try to bid up the stock price by submitting buy orders and simultaneously selling stocks. It turns out that these strategies complicate our existence proofs but we want to consider them since they are not ruled out in most real world trading systems.

dence is the result of a general and realistic set of possible orders such as buy and sell limit orders, market orders, stop orders and all or nothing orders (fill or kill orders).

Market mechanism. By adding up individual behavior an action profile $a \in A$ induces the market excess demand correspondence

$$Z(p) = \sum_{i=0,...,N} Z_i(p)$$

which decomposes into aggregated buy and sell offers

$$D(p) = \sum_{i=0,...,N} D_i(p) \text{ and } S(p) = \sum_{i=0,...,N} S_i(p)$$

called the *market demand* and *market supply correspondences.* They define sets of quantities the market as a whole is willing to buy or to sell at a given sell price p. Most relevant for many real and theoretical market mechanisms is the *limit order trade volume* $\tau(p)$ for $p \in P$ defined as the maximum tradable quantity

$$\tau(p) = \min \left\{ \max \sum_{i=0,...,N} d_i(p), \max \sum_{i=0,...,N} s_i(p) \right\}$$

of the short side of the market restricted to limit orders and stop orders[8] but excluding all-or-nothing orders. The small letters d_i and s_i indicate convex valued demand and supply correspondences composed only by limit orders and stop orders (for details see appendix A on page 27).

Denote by $((x_0, ..., x_N), (y_0, ..., y_N))$ a buy-sell-transaction vector if $x_i, y_i \in \left\{0, \frac{1}{M}, \frac{2}{M}, ..., \frac{M}{M}\right\}$ and $\sum x_i - y_i = 0$. For initial allocation α and net trade vector $x - y$ the final allocation is $\omega = \alpha + x - y$.

We call player i *strictly wealth constrained* iff i can only submit sell orders[9]. For a wealth constrained player i a bid strategy consists only of selling bids.

Next, we define market mechanisms more formally. Intuitively, a market mechanism collects all orders. Given all submitted orders, the market mechanism then announces the price at which trade occurs. Furthermore, the market mechanism determines which orders are executed and who trades.

Definition 1 For any initial ownership $\alpha \in \Delta$ and any strategy profile $a \in A$ a *market mechanism* μ with bid ask spread γ is a function $\mu(\alpha, a) = (p^\mu(a), x^\mu(a), y^\mu(a))$ that picks a sell price $p^\mu(a) \in P$, a buy price $p^\mu(a) + \gamma$ and for any player a subset of submitted orders that are executed, i.e.

$$\begin{aligned} x_i^\mu(a) &\in D_i(p) \cup \{0\} \text{ and} \\ y_i^\mu(a) &\in S_i(p) \cup \{0\}. \end{aligned}$$

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[8]

[9] For example, this is likely to be the very reason why the distinguished player needs funding by outsiders. Otherwise he would prefer to run the firm himself.

We call this latter property *voluntary trade.* Hence, net trades $x_i^{\mu}(a) - y_i^{\mu}(a) \in Z_i(p) \cup \{0\}$ are composed by submitted orders. By specifying the trade vector the market mechanism μ thereby determines the ex post ownership structure given as

$$\omega^{\mu}(a) := \alpha + x^{\mu}(a) - y^{\mu}(a) . \qquad \square$$

Voluntary trade means that *only* submitted orders or nothing are executed, i.e. nobody can be forced to trade and conversely nobody can enforce trade. Market mechanism μ is said to *maximize the trade volume* if μ picks a price that maximizes the limit order trade volume $\tau(p)$. Of course, market mechanisms can be specified that do not maximize the trade volume. For example, price could be determined by maximizing the total trade volume including fill or kill orders. Alternatively the distinguished player or some "important" players could be treated with priority if they submit orders. However, we are not aware of a real world market mechanism not using maximal trade volume with top priority. Nevertheless, our characterization results in section 4 do not rely on this property of maximal trade volume. Further, a market mechanism is called *anonymous* if permuting players' names does not affect price and final allocation.

Market game Γ_{μ}. Any market mechanism μ with bid ask spread γ together with an initial ownership α induces a *market game* Γ_{μ} with strategy space A and *payoff functions* given by

$$u_i(a) = \omega_i^{\mu}(a)\left(\underline{v} + e(\omega^{\mu}(a))\Delta v\right) - \left(p^{\mu}(a) + \gamma\right) x_i^{\mu}(a) + p^{\mu}(a) \cdot y_i^{\mu}(a)$$

for $i = 1, ..., N$ and

$$u_0(a) = \omega_0^{\mu}(a)\left(\underline{v} + e(\omega^{\mu}(a)) \cdot \Delta v\right) - \left(p^{\mu}(a) + \gamma\right) x_0^{\mu}(a) + p^{\mu}(a) \cdot y_0^{\mu}(a) - c(e(\omega^{\mu}(a)))$$

for the distinguished player $i = 0$.

It is instructive to note the difference in the payoff functions between the distinguished player and the outside investors. The distinguished player takes his effort costs $c(e(\omega^{\mu}(a)))$ into account whereas outside investors are only interested in the monetary payoff of their shares. It is due to this asymmetry that the problem we look at is not trivial. Any action profile a in a deterministic market mechanism generates a unique final allocation $\omega^{\mu}(a)$ and thereby induces a unique optimal effort decision $e(\omega^{\mu}(a))$. Hence, company value $v(a)$ is given as

$$v^{\mu}(a) \quad = \quad \underline{v} + e(\omega^{\mu}(a))\Delta v$$

No arbitrage. A strategy profile a of the market game Γ_{μ} is said to satisfy the traditional *no-arbitrage condition* iff there exists no trader i who realizes strictly positive gains under the price $p^{\mu}(a)$ and buy-sell transactions $x_i^{\mu}(a), y_i^{\mu}(a)$ determined by market mechanism μ:

$$[p^{\mu}(a) - v(a)]\, y_i^{\mu}(a) + [v(a) - p^{\mu}(a) - \gamma]\, x_i^{\mu}(a) \leq 0$$

for all $i = 0, \ldots, N$.

Generalized No-arbitrage and Equilibrium. A strategy profile a^* is said to satisfy *generalized no-arbitrage* if it is a *Nash equilibrium* or just *equilibrium* of market game Γ_{μ}, i.e. no player can strictly improve or in the language of game theory every player plays a best response a_i^* to other players strategy profiles a_{-i}^*. Correspondingly, $(p^*, x^*, y^*) = \mu(\alpha, a^*)$ and $\omega^* = \alpha + x^* - y^*$ are called *equilibrium price*, *equilibrium trades* and *equilibrium ex post allocation* of market game equilibrium a^* under market mechanism μ. Furthermore, we will call $e(\omega(a^*))$ equilibrium effort denoted by e^*. While any equilibrium by definition satisfies generalized no arbitrage hitherto it is not clear if the same holds for traditional no-arbitrage.

No Trade Equilibrium. If no player submits an order no player can gain anything by submitting orders in the fully rational setup. This simple observation together with the *voluntary trade property* guarantees that without noise traders for any market mechanism there always exists a trivial no-trade equilibrium where no player submits orders. Since this equilibrium does not offer any clue on stock prices – our main object of interest – we concentrate on more interesting equilibria where we can observe a price such that trade occurs.

True value and excess returns equilibria. An equilibrium a^* with $\omega^* \neq \alpha$ is called *trade equilibrium* of Γ_{μ}. *Excess returns* for a firm are defined as

$$v^{\mu}(a^*) - p^{\mu}(a^*),$$

i.e. the difference between equilibrium firm value and equilibrium price. A trade equilibrium in which shares are traded at their equilibrium value is called a *true value equilibrium*, i.e.

$$v^{\mu}(a^*) = p^{\mu}(a^*) \texttt{ and } \omega^* \neq \alpha.$$

A trade equilibrium in which shares are traded strictly below their equilibrium value is called an *excess returns equilibrium*, i.e.

$$v^{\mu}(a^*) > p^{\mu}(a^*) \texttt{ and } \omega^* \neq \alpha.$$

In an excess returns equilibrium it is more attractive to be a buyer than a seller. A net equilibrium buyer i who buys enough to strictly overcome transaction cost losses gains

$$(v^* - p^* - \gamma)x_i^* - (v^* - p^*)y_i^* > 0$$

and is called *equilibrium winner*. Although the role of its counterpart — the net equilibrium seller — is less attractive it can still be rational if the alternative is low effort of the distinguished player triggering a lower value for all.

3 Trading Rules and Existence.

In this section we show that excess returns equilibria exist while trade at the true value is not an equilibrium. For this we need to consider any potential deviation for every trader. Since market rules determine how the market price and the allocation changes due to a change of submitted orders, we must become very explicit concerning market rules in all their details. In contrast to this, our characterization results hold for general market micro-structures. These characterization results show that the main properties and economic intuition behind the existence results are robust and carry over to a significantly more general class of market microstructures.

To show existence, we concentrate here on electronic call auctions, because 1) we can use the exact and fully specified rules taken from real world trading mechanisms, 2) we do not need to specify the timing as to who trades when and knows what, and 3) these rules are used in the real world and in the literature.[10]

While we limit our analysis of existence on real world call auctions, there is still considerable degree of freedom in the details. Price setting rules are less problematic since the same rules are used in almost all call auction mechanisms on stock exchanges around the world. In contrast, rationing rules regulating allocation and the priority which orders are executed must be treated with great care. Rationing rules differ across exchanges and are relevant for our analysis, in particular rules concerning *size priority* and *fill-or-kill orders*.[11]

Size priority. Size priority is used in different exchanges[12] and in one example we use an opening auction motivated by the Tokyo stock exchange. The trading period in Tokyo

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[10] Trading rules can be ordered by continuous trading vs. call auctions and electronic market places vs. dealer markets. For existence results, one cannot use dealer markets since they are not sufficiently explicit. For example, as a rule specialists on the NYSE "*... have an exchange mandated obligation to maintain fair and orderly markets.*" (Lehmann and Modest (1994, p. 952)). To show existence, it is necessary to overcome the lack of preciseness in the regulation of specialists. In the literature, this problem is often solved by assuming that there is perfect competition between market makers and hence equilibrium price equals equilibrium value. Since a major goal of this paper is to derive the equilibrium trade price endogenously and to allow for a deviation from this assumption, this approach is not feasible here. Moreover, continuous trading is more involved as we would have to specify the timing of orders and the information set for every agent.

[11] Reny and Perry (2006) put similar emphasis on the exact specification of rationing rules.

[12] There are also several exchanges where size priority is not used, for example at the Paris Bourse (see Biais et al. (1995) or Australia stock exchange (see Aitken et al. (1998)).

starts with an opening auction and orders valid for the opening auction can be submitted prior to the trading period. Furthermore,

> ”...all limit orders received prior to the start of trading have equal time priority. Price, time, and size priority hold in this order for limit orders placed during the trading sessions.” (Lehmann and Modest, 1994, p. 954)

Breaking up orders. Different priority rules are applied at NYSE.

> ”The NYSE does not follow a strict time priority rule. To minimize the breaking up of large orders, the time priority rule applies only to the first limit order. The remaining limit orders follow a size priority rule; namely limit orders that match the size of the market order at the best price are given priority over other limit orders ...” (Huang and Stoll, 2001, p. 506)

Accordingly, the objective of not breaking up orders is applied in the market mechanism we call NYSE.[13]

Fill-or-kill orders. Similar to size priority, fill-or-kill orders are allowed at some – e.g. Amsterdam or XETRA – but not all real world call auctions. Note, however, that they can be used in most continuous trading settings, e.g. NYSE or Paris[14].

To establish a realistic setting we define three different market micro-structures which are all call auctions. All three call auctions use the same price setting rules and set price priority as the most important allocation rule. Rationing rules apply if price priority does not already lead to a unique allocation. It is at this point, i.e. the second rule concerning allocation, that these market mechanisms differ.

Call auctions.

A: Price setting.

1. The price is set to maximize the trade volume $\tau(p)$.[15]

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[13] It can be argued that the upstairs market used at NYSE and many other exchanges – e.g. Paris Bourse or XETRA – also gives priority to large orders since only large orders can be traded upstairs (and also downstairs).

[14] See Venkataraman (2001, p. 1450).

[15] Fill or kill orders can be submitted. However, they do not have an impact on price setting. This means that the price and the corresponding executable trading volume or excess demand are calculated as if the fill-or-kill order was not present. A description of the Amsterdam stock exchange (AON are all or nothing orders which is another word for fill or kill orders) as taken from http://www.keytradebank.com/form.html?level=form&option=rul&market=aex is similar: ” on the segment of the double auction, ... the fixing price is calculated without the AON orders. Just before the fixing, the AON orders are added to the orderbook.”

2. Should there be more than one such price, surplus $|s^*(p) - d^*(p)|$ is minimized, not counting fill-or-kill orders.
3. Should there still be more than one potential price, the minimal price will be taken if there is excess supply. For excess demand, the maximum price is taken.
4. Should there still be more than one price, the price closest to a reference price will be chosen and we choose $\bar{v}$ to be the reference price.[16]

B: Allocation rules:

Rule	Amsterdam μ_A	Tokyo μ_T	NYSE μ_N
i.)	Orders are executed according to price priority. This rule does not apply to fill-or-kill orders. Stop orders are not executed.		
ii.)	Fill or kill orders are only matched against each other if they cannot be executed against normal bids. The allocation of fill-or-kill orders maximizes executable trading volume.	Fill-or-kill orders are not executed. Orders with limit price p^* are executed using size priority.	Fill-or-kill orders are not executed. Fully executable orders using p^* as limit price are executed first.[17]
iii.)	Orders with the same priority are executed in a random order.		

We are now in a position to formulate our existence results.[18] The first result is negative. Trade at the true value is not an equilibrium in a fully rational market if trade is organized in an anonymous call market. Our second result is positive. In the same set-up excess returns equilibria exist and we can thereby analyze stock price behavior of firms with a distinguished player using the standard equilibrium concept in a fully rational framework.

Theorem 1 *Consider the market game Γ_μ with sufficiently small tick size δ. Then, the following is true.*

(I) Trade at the true value cannot be an equilibrium under the Amsterdam, Tokyo, and NYSE market microstructure.

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[16] The reference price in real world trading systems is the last traded price (XETRA, p. 27). Since our model only allows for one round of trading, we cannot use the last traded stock price as the reference price.

[17] Think of this rule as follows: Every order on the short side of the market and every order on the long side of the market which does not use p^* as limit price are matched first. From the remaining orders on the long side of the market, an order is drawn from the subset of all executable orders. After this draw has been matched, another order is drawn from the (new) subset of fully executable orders. This procedure is continued until no fully executable order exists on the long side of the market. Then, a draw is taken from all remaining orders that use p^* as the limit price and this order is broken up.

[18] Note that the call auctions we use for existence are stochastic market mechanisms. A more rigorous formulation of stochastic market mechanisms and the induced stochastic market game is postponed to appendix B and will be used extensively in the proofs in appendix C.

(II) However, there exists an excess returns equilibrium under Amsterdam, Tokyo, and NYSE market microstructures for sufficiently small bid ask spread γ. □

Proof is to be found in appendix C, page 29ff.

The first part of theorem 1 shows that trade at the true value is inherently instable since it disregards fundamental economic incentives. The distinguished player always wants to sell shares. If effort is continuous, selling one single share already yields an improvement for the distinguished player. Under the same conditions, however, excess returns equilibria exist. In other words, trading at a price strictly below equilibrium value is consistent with economic incentives.

We have already seen in the introductory example that excess returns, i.e. the gap between trade price and the true value depends on effort costs and can be substantial in equilibrium. It is also straightforward to construct equilibria with substantial excess returns in this general setup. For example, under the Amsterdam market microstructure it suffices if the distinguished player is the only seller who submits a fill or kill order to sell a large number of shares. The limit price of this critical order can then be substantially below its equilibrium value. While outside investors would be most happy to buy from other outside investors at this price they risk to loose the entire value enhancing contribution of the distinguished player if he succeeds to sell his whole package.

The proof of the first part proceeds by characterizing any potential candidate true value equilibrium and finding a contradiction. In every candidate true value equilibrium the following is true. Either the distinguished player has an incentive to change his ex post holding and adjust effort accordingly. Or outside investors have an incentive to change their ex post holdings to trade less against the distinguished player. The driving force to rule out any stochastic ex-post ownership for the distinguished player is that outside investors and the distinguished player can never be indifferent at the same time between buying and selling due to different valuations for shares of the firm. Anonymity of our call auctions makes sure that outside investors or the distinguished player can indeed deviate and change ex post ownership ω. It is important to note that our rigorous proof in Appendix C does not rely on pure strategies although already the formulation of pure strategy spaces is complex as we show in appendix A.

The easiest way to prove part two – existence of an excess returns equilibrium – is to construct a particularly simple such equilibrium. Clearly, there are many possibilities for such equilibria and more realistic equilibria are typically more involved. The intuition for our proof of the second part follows the same idea outlined already in the introductory example in section 1.

4 Trade Equilibrium Characterization

Having shown existence for some specific but relevant trading rules, we continue with the more general results, the characterization of excess returns equilibria.

Theorem 2 *Let a^* be an excess returns equilibrium. Then, for any market mechanism μ the following properties are satisfied.*

1. *In equilibrium a^* the distinguished player exerts effort $e^* > 0$.*
2. *In equilibrium a^* the distinguished player $i = 0$ is not a seller $\omega_0 \geq \alpha_0$.*
3. *Each investor is pivotal in the sense that selling less than specified or buying more than specified by equilibrium strategies triggers the distinguished player $i = 0$ to sell more or buy less and to reduce effort subsequently or it triggers a price increase.*
4. *The distinguished player submits an order.*
5. *In an excess returns equilibrium with an equilibrium winner traditional no arbitrage does not hold while generalized no arbitrage holds by definition.* □

Proof to be found on page 34.

No distinguished player. This formulation of the model contains the special case $\Delta v = 0$ with no distinguished player. The following proposition shows that models without distinguished players have no excess returns equilibria and therefore are not robust with respect to the introduction of arbitrarily small distinguished players $\Delta v > 0$ if excess returns equilibria exist.

Proposition 1 *For a model without a distinguished player $\Delta v = 0$ excess returns equilibria do not exist and therefore traditional no arbitrage is always satisfied.* □

Proof to be found on page 34.

Putting the results in context. We combine a standard *asset pricing* setup with a principal-agent moral hazard problem. The latter is a widely accepted theoretical tool in *corporate finance.* In our merged framework of these two classical perspectives in finance we have shown that *excess returns equilibria* turn up as the only rational and consistent mode of behavior. One way of looking at this modelling exercise is that by adding a distinguished player to a market game the game turns from a 0-sum to a non-0-sum game.[19] Another way to interpret this is that by adding a distinguished player introduces an externality into a competitive market. It is well known from general equilibrium analysis that this change in the model removes one of the crucial assumptions for the fundamental welfare theorems about efficiency. Against this background we should not

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[19] It is well known since Nash's extension of v.Neumenn-Morgenstern's equilibrium-definition for 0-sum games that turning a game from a 0-sum to a non-0-sum game has many important consequences. Meanwhile there is a wide consensus in the profession that Nash's definition played a crucial role in triggering the enormous rise of game theory as accepted method in economic theory.

expect from financial markets to yield an efficient result in our context — i.e. if a distinguished player matters.

In the following section we analyze the robustness of excess returns equilibria when irrational noise traders are introduced and the number of outside investors is allowed to be arbitrarily large.

5 Noise and Small Price Takers

We have seen in proposition 1 that excess returns equilibria do not exist without a distinguished player. In this section we show that other seemingly salient ingredients of the rational and strategic model in fact are not crucial. The additional liquidity provided by irrational traders is even helpful to come up with another explanation for the excess returns phenomenon. Pivotalness or strategic influence on the price is not necessary anymore. Mathematically, the least form of strategic influence on prices by rational traders is given by a continuum of traders. The logic is the following. By some traders' irrationality the price and final allocation become random variables. Moreover, irrationality raises the chances for rational investors to gain by buying from other traders than the distinguished player. This includes the possibility that in this equilibrium small rational investors can behave as price takers and still gain in expectation. These traders face the following trade-off. To raise the buying price limit above the equilibrium trade price raises the chances to buy for an attractive price below the equilibrium value. However, the downside is that this strategy makes it also more likely to trade against the distinguished player at some higher price that in turn happens to exceed equilibrium value.

Specifications. The set of investors in this section is given by

$$i \in I = [0,1] = DP \cup RI \cup NT = \{0\} \cup (0,1) \cup \{1\}$$

consisting of three types of investors. As before the distinguished player is $i = 0$ and small rational outside investors are $i \in (0,1)$. The distinguished player initially owns proportion $\alpha_0 \geq 0$ of shares and rational outside investors together own $\alpha_r < 1 - \alpha_0$ shares. Now we specify effort costs as $c(e) = c \cdot e^2$. In this section moreover we suppose the presence of irrational noise traders trading for exogenous reasons. Since only their aggregated behavior matters for rational investors they are treated from here as if they were a single irrational investor $i = 1$. These noise traders initially own together the remaining $\alpha_1 = 1 - \alpha_0 - \alpha_r$ shares.[20]

The distinguished player is assumed to be strictly wealth constrained. To make sure that best responses are well defined we further assume that every rational investor is

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[20] More formally, initial ownership structure $\alpha \in \Delta$ in this section is a measure with $\int_I \alpha_i di = 1$. Outside investors are small investors who individually own 0 and only jointly own a strictly positive fraction of shares.

budget constrained with a finite budget $B_i < \infty$. The aggregated budget constraint across all rational investors is non-binding and larger than $\bar{v}$ meaning that jointly outside investors can afford to buy the entire firm even at the highest reasonable price.

Noise. Suppose noise traders only submit market orders and hence only the excess demand correspondence of noise traders denoted by $\tilde{Z}_1$ matters for the rest of the market. We further suppose that $\tilde{Z}_1$ is a random variable with support $[-\alpha_0, b] \subset \mathbb{R}$ where $-\alpha_0 < 0 < b \leq \alpha_r$. The assumption $b \leq \alpha_r$ means that the event $\tilde{Z}_1 > \alpha_r$ that noise traders want to buy more than rational investors own has probability 0. We introduce this assumption to make sure that existence of excess returns equilibria is not driven by the specification of noise. The distribution function F is assumed to be continuous and symmetric. In particular, $\Pr(\tilde{Z}_1 = 0) = 0$ and $F(0) = \frac{1}{2}$ meaning that the events $\Pr(\tilde{Z}_1 > 0) = 1 - F(0) = \frac{1}{2}$ and $\Pr(\tilde{Z}_1 < 0) = F(0) = \frac{1}{2}$ are equally likely.[21]

Theorem 3 *Consider sufficiently small tick size $\delta > 0$ and bid-ask spread $\gamma = 0$. Then,*

(i) for any non-degenerate symmetric noise F, there exist initial ownership structures α and effort cost parameter c such that an excess returns equilibrium exists under the NYSE market microstructure μ_N,

(ii) for some non-degenerate symmetric noise F, there exist initial ownership structures α and effort cost parameter c such that an excess returns equilibrium exists under the Tokyo market microstructure μ_T

(iii) Without noise, an excess returns equilibrium cannot exist with a continuum of traders under any call auction mechanism.

(iv) Without a distinguished player ($\bar{v} = \underline{v}$) excess returns equilibria do not exist under the NYSE or Tokyo market microstructure under any non-degenerate distribution F. □

Proof to be found on page 34.

Again, it should be noted that parameters can easily be specified such that excess returns can be substantial. Furthermore, for any noise, excess returns equilibria exist for a whole range of cost parameters and for every cost parameter it holds for a whole range of ownership structures. Since the crucial conditions are strict inequalities a similar proof works for sufficiently small bid-ask spread $\gamma > 0$.

The following line of arguments provides the main intuition of the proof and shows why investors have no incentive to bid up the share price. In our excess returns equilibria with noise, the distinguished player sells his shares with strictly positive probability whenever

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[21] The symmetry condition is sufficient but not neccessary. A weaker sufficient but more technical condition is to assume that the probability that the distinguished player can sell his entire stake against noise traders is not too small and not too large.

$p \geq p_0$. Shares are overvalued at these high prices which implies that $p_0 \in (\underline{v}, \bar{v})$. However, for any price $p \leq p_0 - \delta$, the distinguished player does not sell his shares. As a result, shares are undervalued if the resulting price is $p \leq p_0 - \delta$. Hence, rational investors want to sell if $p \geq p_0$ and to buy if $p \leq p_0 - \delta$. This implies that the value of the firm is price dependent and investors are always rationed: There is excess demand if $p \leq p_0 - \delta$ and excess supply if $p \geq p_0$. Principally, rational investors can overcome the rationing by increasing their buy limits. The downside from this strategy is that they always buy shares, which are sometimes undervalued and sometimes overvalued.

Noise traders in this setting are important in two respects; i) noise traders make the price stochastic and ii) the distinguished player sells his shares to noise traders with positive probability. Concerning the first aspect, excess demand from noise traders is stochastic and hence prices are stochastic. With positive probability, the high price p_0 occurs and with positive probability the low price $p_0 - \delta$ occurs. As a result, outside investors do not know in advance whether or not the high or the low price will be realized. Outside investors react to this uncertainty by making use of limit orders. They are willing to buy shares at any price (weakly) below the low price. And outside investors are willing to sell shares whenever the price is (weakly) above the high price. Recall that the shares are undervalued conditional on the fact that the distinguished player does not sell his shares and overvalued conditional on the fact that the distinguished player does sell his shares. This implies that the extent of excess returns depends on the likelihood that the distinguished player sells his shares. For symmetric noise which we consider in the proposition, the undervaluation part dominates. This is true for the following reason. Under symmetric noise, the likelihood of observing the high and the low price is each equal to one half. The distinguished player never sells his shares at the low price and hence shares are always undervalued at the low price. If the high price is observed, the distinguished player does not sell his shares all the time - the distinguished player may be rationed. This rationing implies that on average shares will be undervalued and we observe excess returns. Note that this also implies that with very asymmetric noise and large buying pressure from noise traders, shares may be overvalued on average.

We now turn to the second role of noise traders. Noise traders are those investors who actually buy from the distinguished player. This does not happen all the time when the high price p_0 occurs but it happens with positive probability. Hence, an important assumption is that excess demand from noise traders can be high enough to buy out the distinguished player. In this sense, noise traders provide liquidity. They make it possible that the distinguished player can sell his shares - not always but at least from time to time.

It is interesting to compare the noisy environment with our earlier analysis. First of all and most important, our voluntary trade property of market mechanisms no longer holds. Noise traders are irrational and are forced to trade for exogenous reasons.

Since the voluntary trade property does not hold for all investors our characterization results of section 4 are not valid any more. In particular, *pivotalness* ceases to hold. In the voluntary trade framework, pivotalness was the key to produce liquidity. Investors had an incentive to trade in equilibrium to guarantee that the distinguished player does not sell his shares. In the present context, the role of providing liquidity is taken by the noise traders. Therefore, excess returns equilibria exist even if investors are small price takers who are not pivotal.

Since the rationing factor is determined by comparing noise against rational investors, an increase of noise facilitates the existence of excess returns equilibria for two reasons. First, liquidity is increased which makes it more likely that the distinguished player can sell his shares on the market. Secondly, the rationing problem is reduced which implies that rational investors have a smaller incentive to increase their limit price used in their buy orders.[22]

6 Related Literature

The paper relates to empirical and theoretical contributions, in particular those jointly addressing corporate governance and asset pricing. We will first discuss the empirical literature and argue that there is evidence for i) excess returns equilibria, ii) the existence of distinguished players and iii) non-atomistic, pivotal investors with price impact. Second, we discuss related theoretical contributions.

Empirical literature. In order to support the *excess returns equilibrium* phenomenon formulated by this theory, one has to identify a distinguished player. As potential candidates, von Lilienfeld-Toal and Rünzi (2007) investigate CEOs who own a non-negligible fraction of shares. They show that this presence of an owner-CEO is not priced. They consider the universe of S&P 1500 (S&P 500) firms from 1996-2005 (1994-2005) and find that a portfolio consisting of S&P 1500 firms in which the CEO owns more than 10% of shares of the company produces statistically significant annualized abnormal returns of approximately 12%. These results carry over to several robustness checks including a formulation of industry adjusted returns and splitting the sample into boom and burst years. Moreover, von Lilienfeld-Toal and Rünzi (2007) investigate to which extent excess returns differ if effort of the CEO is important. To do so, they sort firms with regard to managerial discretion. They find that excess returns tend to be more important for firms in industries where the CEO has a strong influence on firm performance,

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[22] More precisely, we can show that excess returns equilibria are more important when noise increases in the following twofold sense. Firstly, the maximum quantity of excess returns that can be supported in an excess returns equilibrium increases. Furthermore, the measure of the set of parameters increase that support excess returns equilibria. Note, though, that these statements apply to the specific class of excess returns equilibria we construct to show existence in theorem 3.

younger firms, and growth firms. Other potential candidates for distinguished players are founder-CEOs. Fahlenbrach (2009) finds that founder-CEO firms outperform the market by approximately 10% and these results are again robust to various specifications. While these results are consistent with excess returns equilibria, they are inconsistent with true value equilibria. Hence, true value equilibria may not only fail to exist in theory, empirical evidence also suggests that excess returns equilibria are more relevant, provided good candidates for distinguished players are found.

Aforementioned papers also document the empirical importance of *distinguished players* and value increasing shareholders for listed US firms. For example, within the S&P 1500 firm universe, according to von Lilienfeld-Toal and Rünzi (2007) more than 10% of firms have an officer who owns more than 10% of outstanding stocks. Fahlenbrach (2009) reports that founder-CEOs are present in 11% of the largest US firms (founders hold on average 11% of shares of a firm). A similar emphasis is put forward in the recent paper by Holderness (2006). In a representative sample of 375 US firms he reports that 96% of US firms have at least one blockholder who owns more than 5% of shares of the firm. Average ownership of all blockholders, directors, and officers is 43% (median 43%), average ownership of the largest shareholder is 26% (median 17%), and average ownership of officers and directors is 24% (median 17%). The latter finding is consistent with results in Fahlenbrach and Stulz (2007) who look at the much larger universe of US firms covered in the compact disclosure discs. They analyze $27,636$ firm years from 1988-2003 and report mean ownership of officers and directors to be 22.4% (median 15.8%).

One group of shareholders that are reasonable candidates for non-price taking, pivotal outside investors are *institutional investors* who control more than 100 million US dollars. It is well documented that trades of this group of investors have an influence on the price. Chan and Lakonishok (1995, p. 1147) argue that "*For many institutional investors, however, even a moderately-sized position in a stock may represent a large fraction of the stock's trading volume*". They document an average price impact of 1% for buy orders or $-.35\%$ for sell orders. Their sample consists of NYSE and AMEX trades of 37 large investment management firms from July 1986 until the end of 1988. Noteworthy, the trades of these 37 institutional investors accounted for approximately 5% of trading volume on NYSE and AMEX in this time period.

Apart from the importance of trading volume of institutional investors, it is also known that the ownership of institutional investors is economically significant. Gompers and Metrick (2001) consider the holdings of institutional investors from 1980-1996. Shareholdings of institutions is increasing over time and in December 1996, the last quarter of their sample, institutional investors hold more than 50% of the market capitalization of US firms. We interpret these observations that there are only a few important institutional investors as supportive for the assumption that outside investors can act strategically rather than as pure price takers. In December 1996, there are only 1303 institutions. In particular, the largest 100 institutions hold approximately *one third* (37.1%) of the entire market capitalization and the largest 10 institutions hold 14.6% of market capitalization of all US firms.

Theoretical literature. Most of the theoretical literature about large shareholders and trading games only analyzes what we call *true value equilibria.* Prominent examples include Shleifer and Vishny (1986), Admati, Pfleiderer and Zechner (1994), Maug (1998), DeMarzo and Urosevic (2006), Kahn and Whinton (1998), or Magill and Quinzii (2002). All these papers study a large and value increasing shareholder who may increase a firm's value while increasing a firm's value causes private effort costs. Some of them are more general in other important respects (asymmetric information, dynamic framework, ...) while our paper is more general with respect to the market microstructure.

With respect to *excess returns equilibria* closest to our paper is von Lilienfeld-Toal (2005), who identifies an excess returns equilibrium for one specific market microstructure and then turns attention to the empirical implications of excess returns. In particular, it argues that excess returns equilibria are consistent with i) negative abnormal returns around unlock days and ii) positive abnormal returns for firms with a distinguished player. In contrast to our paper it does not show that trade at the true value is not an equilibrium, it contains no general characterization results, and does not consider irrational traders. Note that excess returns equilibria may also occur in the model of Bolton and von Thadden (1993) which is concerned with corporate control issues. In contrast to this article it does not focus, however, on (asset) pricing implications of excess returns equilibria. In particular, it does not relate excess returns to no-arbitrage in asset pricing. Rather, they are mainly interested in the question when blocks of shares remain, vanish or are newly created. The reason as to why excess returns equilibria may exist in the model of Bolton and von Thadden (1993) is similar to our notion of pivotalness. A related explanation of takeovers is given by Bagnoli and Lipman (1985) or Holmstrom and Nalebuff (1992). The latter papers analyze potential solutions to the free rider problem first mentioned by Grossman and Hart (1980). [23]

All above mentioned papers derive the results using a particular *market microstructure.* We are not aware of other papers deriving characterization results that are valid for general market micro-structures. Conversely, our assumptions on complete symmetric information in a static market game are clearly more specific in other respects than many of the above mentioned contributions.

Our setting can be viewed as double sided auctions with strategic trading and the paper relates to this branch of *market microstructure theory.* Papers falling within our framework are for example Kyle (1985), Kyle (1989), Rochet and Vila (1994), or Reny and

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[23] In one sense, our model could be interpreted as a generalization of Bagnoli and Lipman (1985) and Holmstrom and Nalebuff (1992) if the distinguished player's value-enhancing capability only unfolds for $\alpha_0 \geq \frac{1}{2}$, the strategy space of the distinguished player is limited to a takeover bid, and other shareholders can only submit sell orders.

Perry (2006). While their exact specification of price setting and quantity allocation rules is within our class of market mechanisms, the economic environment we are interested in is distinct as compared to these papers. Moreover, market microstructure theory is also interested in the price impact of individual trades which is aptly pointed out by O'Hara (2003): "... asset pricing ignores the central fact that market microstructure focuses on: Asset prices evolve in markets".

The present article also relates to the literature on *no trade theorems*, for example Milgrom and Stokey (1982) or Tirole (1982). The driving force behind no trade theorems is the fact that there are no gains from trade or negative gains from trade in the presence of transaction costs. In the class of models we are interested in, gains from trade are zero for true value equilibria and consequently, true value equilibria in the traditional sense fail to exist for positive bid ask spreads. In excess returns equilibria, in contrast, gains from trade are no longer zero sum since the owner manager's threat to sell is viable and trade at a low price prevents the owner manager from selling. Further, as in the no-trade theorem literature our continuum-trader-version shows that noise is needed to initiate trade and provide liquidity.

Further, the paper relates to the vast literature on agency problems as in Holmstrom (1979) or Grossman and Hart (1983). In particular, models with bilateral contracting and non-exclusive contracts are concerned with externalities among trading partners. Examples are Bisin and Guaitoli (2004), Bizer and DeMarzo (2004), Kahn and Mookherjee (1998) and Segal and Whinston (2004). In these papers, a distinguished player can write contracts with many players while in our model, the distinguished player can anonymously trade with many outside investors.

Finally, our model relates to the literature on *large games* since it establishes a prime example of a *semi-anonymous game* where players' payoffs only depend on aggregated actions of player-types rather than on the entire individual action profile. In this context we only consider two types of players, regular investors and the distinguished player. Now, *semi-anonymity* is a critical property since it means that traders do not care about the composition of bids among regular shareholders. However, being of a different type, the distinguished player's actions impose externalities and distinctly enter the preferences of other traders. Equilibria of semi-anonymous games in general have been characterized in Blonski (2005). Strong theoretical support for their relevance provides Kalai (2004) who shows that equilibria of semi-anonymous games are robust with respect to the extensive form of the underlying game when the number of players gets large. This result is relevant and important for markets with continuous time trading where it is generally hard to know on which information traders base their decisions. With few traders for any kind of strategic trading game theorists would expect that traders' information affects the outcome substantially. Kalai shows, however, that equilibria of any extensive form are approximately ex-post Nash — i.e. deviation incentives get arbitrarily small — if the number of players gets large. Since the deviation incentives in our non-existence proof for true value equilibria are substantial Kalai's result gets powerful in this context since it implies that we can neither expect true value equilibria to exists in dynamic versions for this model including all kinds of information asymmetries nor in continuous time trading markets.

7 Extensions

It is straightforward to set up a similar theory with more than one distinguished player $j = 0, ..., M$ each of them characterized by his stake, effort cost and potential value $\alpha_j, c_j, \Delta v_j$. Clearly, since distinguished players can behave as regular investors as well the presence of a single distinguished player is sufficient to raise the possibility of excess returns equilibria. For empirical testing we expect that the significance of the most important distinguished player is a better predictor for excess returns than the number of distinguished players since triggering this player to sell causes the biggest potential payoff losses. If the market price offers the right incentives for this most important distinguished player it should as well do for minor distinguished players.

We have not discussed the potential role of market makers in our model even though our market microstructure allows for an analysis of market makers. Analyzing the role of market makers can be done as follows. Suppose agent $N+1$ is the market maker. Moreover, agent $N+1$ submits buy and sell orders $(1, \infty)$ and $(1, -\infty)$. In other words, the agent $N+1$ submits to buy and sell the entire firm. Hence, any market imbalance can be bought by agent $N+1$ or sold to agent $N+1$ without violating our "voluntary trade" assumption. Moreover, the market maker is hardwired such that he maximizes u_{N+1}, potentially subject to institutional rules such as market clearing, i.e. offering a clean up price. The resulting allocations and prices will be the same as in a market mechanism in which all agents $i = 0, 1, ..., N$ submit their shares, a market maker observes these orders and sets a price to maximize his utility, subject to obeying institutional rules.

Since our model in principle exhibits multiple equilibria, empirical observations might help to judge which equilibria are most relevant in reality. Building on this observation, the following questions could be addressed in future empirical research: How general is the occurrence of excess returns equilibria for smaller firms than S&P1500? How do abnormal returns increase or decrease with firm size and with the initial stake of the distinguished player? Can we detect the importance and magnitude of private costs of control from these excess returns equilibria? Are there any differences in trading volume for firms with and without a distinguished player, especially if we control for transaction costs? Moreover, it might be possible to test more general aspects of game theory using stock market data and the methodology of games in aggregated form developed in Blonski (2005) which allows robust predictions on the structure of large semianonymous games without specific knowledge about individual preferences. The numerous implications of excess returns equilibria promise further interesting combinations of theoretical and empirical work on incentives, game theory and asset pricing.

In earlier versions of the paper we included general formulations of the model and its crucial results for discrete effort choice $e \in \{0, 1\}$ as in the introductory example, continuous price and quantity specifications, and existence of excess returns equilibria for other market mechanisms extending further the generality of excess returns equilibria.

Our model is a simultaneous and static model which does not incorporate dynamic aspects of real world continuous time markets. Therefore, it is important to see how this theory translates into a dynamic version. Further, information is typically asymmetric if distinguished players trade in stock markets. Although it is known from game theory — as mentioned before in section 6 in the context of Kalai's (2004) large robust games — that the static full information case is most relevant for the structure of incomplete information settings in particular if the number of traders gets large it is nevertheless an important, interesting and challenging project to study the interplay between the structure of excess returns equilibria with informational asymmetries. As the notion of no-arbitrage equilibrium in traditional capital market valuation theory generalizes from the complete information case to the case of information asymmetries we conjecture that the notion of generalized no-arbitrage as formulated in this article generalizes as well to the case with informational asymmetries.

8 Conclusions

We show that shares of a firm with a distinguished player cannot be priced correctly. Trading at the true value is not consistent with incentives and rational behavior if the market is anonymous. In contrast, excess returns equilibria exist in both, a fully rational world and a world with noise and a continuum of traders. We characterize excess returns equilibria for a general class of market mechanisms. It turns out that the existence of a distinguished player is necessary for excess returns equilibria to exist.

Our theory is general in the sense that it contains the benchmark case of a frictionless efficient market without distinguished player and without transaction costs and with the usual true value equilibria as the special case $\Delta v = \bar{v} - \underline{v} = 0$. In this sense our results show that pricing predictions of models with frictionless markets are not robust with respect to the introduction of an arbitrarily small[24] distinguished player.

The main intuition behind the existence of excess returns equilibria is as follows. Whenever share prices of a firm exceed a certain threshold, the distinguished player prefers to sell his shares – or does not want to buy shares – and reduce effort subsequently. As a result, shares are traded below this threshold price. Due to the private effort costs, this threshold price is below the equilibrium value.

Now, trade can occur for two reasons. Equilibrium sellers – selling shares below

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[24] The size of the distinguished player is defined by his maximal contribution to the company's fundamental value.

the equilibrium value – can be pivotal and highly rational. They then know that not selling shares will trigger the distinguished player to sell shares instead, cut back on the costly effort and reduce firm value. This renders everyone worse off, including deviating equilibrium sellers. In the absence of pivotal traders, noise traders may fill the liquidity gap. Noise traders may sell for exogenous and stochastic reasons. Then, the equilibrium share price is stochastic and the distinguished player has an incentive to sell at the high realizations of the share price but not at the low realizations of the share price. Rational buyers may then prefer not to buy at high share prices but only buy at low share prices. As a consequence, they are not increasing demand – by submitting bids with higher buy limits – and hence they are not increasing the share price even though on average, shares are traded below the equilibrium value.

Additional research questions arise naturally. For example, what happens in a dynamic formulation, under asymmetric information, or with risk aversion? While the economic intuition behind our results is quite strong, it is also apparent that a rigorous formulation of such questions is not straightforward at all. In fact, all our proofs turn out to be involved and full of details.

Since there are two different explanations for trade to occur in an excess returns equilibrium (fully rational, pivotal players or irrational noise traders), it would be interesting to empirically account for the importance of each explanation. Even though there exists some evidence for excess returns equilibria (von Lilienfeld-Toal and Rünzi (2007) and Fahlenbrach (2009)), it is a worthwhile task to identify excess returns equilibria in other circumstances or conversely to identify circumstances where excess returns do not exist.

Investigating different aspects and puzzles of asset pricing, both in theoretical and empirical work also promises to be fruitful. The importance to carefully investigate asset pricing phenomena in light of this theory becomes clear when it comes to judging the empirically observed excess returns. Observing abnormal returns due to a certain investment strategy, as documented by von Lilienfeld-Toal and Rünzi (2007), need not be a sign of irrational behavior but might be the result of excess returns equilibria and highly rational behavior.

9 Appendix A: Orders and Strategies

Orders. Demand and supply correspondences can be composed by sets of orders. The following description of the relevant market actions denoted as *orders* is chosen to model real world market mechanisms as closely as possible. First, denote by $B = Q \times P$ the space of *buy limit orders* with typical element $\beta = (b, p)$ where $p_b = p + \gamma \in P$ denotes the limit price up to which a player is willing to buy any quantity $q \leq b \in Q$. Conversely, for a *sell limit order* $\sigma = (s, p) \in S = Q \times P$ the price p is the minimal price from which the submitting trader is willing to sell q up to quantity s. Buy and sell orders can be interpreted as downward sloping step correspondences. For example, the buy limit order $\beta = (b', p')$ is precisely defined by the correspondence $\beta : P \to Q$ where

$$\beta(p) = \begin{cases} \{q \in Q \,|q \leq b'\} & \text{for } p \leq p' + \gamma \\ 0 & \text{otherwise} \end{cases} .$$

A trader who submits $\beta = (b, \infty)$ or $\sigma = (s, -\infty)$ is said to submit a *market order* since a certain quantity is ordered for buy or sell independently of price. Market order correspondences are bounded by vertical lines. Similarly, the graphs of *buy stop orders* denoted by $\chi = (b_{st}, p)$ and *sell stop orders* denoted by $\psi = (s_{st}, p)$ are upward-sloping step correspondences. The interpretation is that any quantity up to b_{st} is bought above price p or quantity s_{st} is sold below price p. We also allow so called *fill or kill orders* or *all or nothing orders* that specify that a certain quantity is to be bought or sold entirely or not at all. A fill or kill order is denoted by $\mathring{\beta} = (b, p)$, $\mathring{\sigma}$, $\mathring{\chi}$, or $\mathring{\psi}$. More precisely, say for $\mathring{\beta} = (b', p')$, the related correspondence $\mathring{\beta} : P \to Q$ has a non-convex graph and is defined as

$$\mathring{\beta}(p) = \begin{cases} \{0, b'\} & \text{for } p \leq p' + \gamma \\ 0 & \text{otherwise} \end{cases} .$$

Strategies. A *market game strategy* a_i of player $i = 0, ..., N$ is a collection of orders

$$\begin{aligned} a_i \quad = \quad & \{(\beta_i^1, \beta_i^2 ...), (\mathring{\beta}_i^1, \mathring{\beta}_i^2, ...), (\sigma_i^1, \sigma_i^2, ...), (\mathring{\sigma}_i^1, \mathring{\sigma}_i^2, ...), \\ & (\chi_i^1, \chi_i^2 ...), (\mathring{\chi}_i^1, \mathring{\chi}_i^2, ...), (\psi_i^1, \psi_i^2, ...), (\mathring{\psi}_i^1, \mathring{\psi}_i^2, ...)\}. \end{aligned}$$

Denote by A_i the corresponding *strategy space* of player i and by $A = A_0 \times \cdots \times A_N$ the strategy profiles. Adding up buy and sell orders for some player i yields the individual excess demand correspondence

$$\begin{aligned} Z_i(p) &= z_i(p) + \mathring{z}_i(p) \text{ composed by} \\ z_i(p) &= \sum_{\beta, \sigma, \chi, \psi \in a_i} \beta(p) + \chi(p) - \sigma(p) - \psi(p) \text{ and} \\ \mathring{z}_i(p) &= \sum_{\mathring{\beta}, \mathring{\sigma}, \mathring{\chi}, \mathring{\psi} \in a_i} \mathring{\beta}(p) + \mathring{\chi}(p) - \mathring{\sigma}(p) - \mathring{\psi}(p) \end{aligned}$$

adding up buy and sell orders[25] of player i. A market game strategy can be decomposed into buy orders and sell orders. Denote by

$$\begin{aligned} D_i(p) &= d_i(p) + \mathring{d}_i(p) = \sum_{\beta \in a_i} \beta(p) + \sum_{\chi \in a_i} \chi(p) + \sum_{\mathring{\beta} \in a_i} \mathring{\beta}(p) + \sum_{\mathring{\chi} \in a_i} \mathring{\chi}(p) \\ S_i(p) &= s_i(p) + \mathring{s}_i(p) = \sum_{\sigma \in a_i} \sigma(p) + \sum_{\psi \in a_i} \psi(p) + \sum_{\mathring{\sigma} \in a_i} \mathring{\sigma}(p) + \sum_{\mathring{\psi} \in a_i} \mathring{\psi}(p) \end{aligned}$$

player i's individual demand and supply correspondences given as quantities player i is willing to buy or to sell at a given price p. In particular $d_i(p)$ and $s_i(p)$ specify individual demand and supply excluding fill or kill orders.

10 Appendix B: Stochastic Market Mechanisms

Stochastic trade equilibria. Real world market mechanisms often are specified by a list of rules with decreasing order of priority. Sometimes there remains some ambiguity with respect to equilibrium

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[25] It is necessary to differentiate in our notation the cases including and excluding fill or kill orders since in most real world market mechanisms kill or fill orders are treated differently. For example, they are not written in the order book and thereby have no direct influence on the market price.

price or allocation if all rules are satisfied by more than one price and/or set of executed orders such that a random choice may be implemented. In this section we consider risk neutral investors facing a stochastic market mechanism.

A stochastic ownership structure $\tilde{\xi} \in \tilde{\Delta}$ is an element of the space of probability measures $\tilde{\Delta}$ on simplex Δ. Accordingly, define stochastic market prices $\tilde{p} \in \tilde{P}$ and stochastic trade vectors $(\tilde{x}, \tilde{y}) \in \tilde{\Phi}$.

Definition 2 For any initial ownership $\alpha \in \Delta$ and strategy profile $a \in A$ a stochastic market mechanism $\tilde{\mu}$ with bid ask spread γ is a mapping

$$\tilde{\mu} : \Delta \times A \rightarrow \tilde{P} \times \tilde{\Phi}$$

where for initial ownership α and strategy profile a the market mechanism $\tilde{\mu}(\alpha, a) = (\tilde{p}, \tilde{x}, \tilde{y})$ picks a stochastic sell price $\tilde{p}$ (and buy price $\tilde{p} + \gamma$) and for any player trade is voluntary. This means that only submitted orders can be executed, i.e. for any state of nature $(\tilde{x}_i, \tilde{y}_i) \in D_i(p) \times S_i(p)$ and therefore $\tilde{\omega}_i - \alpha_i \in Z_i(p)$. Again, the stochastic trading volume is

$$\tilde{\tau}_{\mu} : \Delta \times A \rightarrow [0, 1]$$

and $\tilde{\tau}_{\tilde{\mu}}(\alpha, a) = \sum_{i=0}^{N} \tilde{x}_i (\alpha, \gamma, a)$. □

The distinguished player picks his effort decision e after the stochastic market game is over and the realizations of all random variables are known. Denote by $\tilde{e}$ the random effort decision induced by the realization of $(\tilde{x}, \tilde{y})$ which determines the final stake of the distinguished player. Similar as before, a stochastic market mechanism $\tilde{\mu}$ together with an initial ownership α and bid ask spread γ induces a *stochastic market game* Γ_{μ} with strategy space A and risk neutral *payoff functions* given by

$$\begin{aligned} u_0(a) &= E\left[\tilde{\omega}_0 \underline{v} + \tilde{e}\left(\tilde{\omega}_0 \Delta v - c\right) - \left(\tilde{p} + \gamma\right) \tilde{x}_0 + \tilde{p} \tilde{y}_0\right] \text{ and} \\ u_i(a) &= E\left[\tilde{\omega}_i \left(\underline{v} + \tilde{e} \Delta v\right) - \left(\tilde{p} + \gamma\right) \tilde{x}_i + \tilde{p} \tilde{y}_i\right] \text{ for } i = 1, ...N \end{aligned}$$

where E means expectation value.

Stochastic true value and excess returns equilibria. A *stochastic true value equilibrium* is an equilibrium where $E\left(p^* | \alpha \neq \omega\right) = E\left(\tilde{v} | \alpha \neq \omega\right)$ and a *stochastic excess returns equilibrium* is defined as an equilibrium where $E\left(p^* | \alpha \neq \omega\right) < E\left(\tilde{v} | \alpha \neq \omega\right)$.

11 Appendix C: Proofs

PROOF (OF THEOREM 1, PAGE 15) (I) Trade at true value is not an equilibrium, (II) existence.

(I) Suppose a^* were a true value equilibrium with $Ev\left(\omega_0^{\mu}(a^*)\right) = p^*$. The proof proceeds by showing that the distinguished player $i = 0$ or some outside investor $i > 0$ can always improve which is a contradiction to a^* being an equilibrium. The logic of the proof is always as follows: Whenever shares are traded in a true value equilibrium, shares are overvalued from the perspective of the distinguished player and he wants to sell. Whenever shares are priced correctly from the perspective of the distinguished player they are undervalued from the perspective of outside investors. To avoid introducing ugly notation for mixed strategies in trading we offer a proof does not rely on pure strategies. We do this by constructing payoff increasing deviations for deterministic *and* for stochastic ex-post ownerships.

The proof uses the following auxiliary result that the distinguished player wants to change his ex post holdings at the equilibrium price. This is shown by proving the following lemma.

Lemma 1 *Consider a candidate true value equilibrium* a^* *with corresponding* ω^*, x^*, y^* *and deterministic* ω_0^**. Then, strategy* a_0' *generating profile* $a' = (a_0', a_1^*, a_2^*, \ldots, a_N^*)$ *is a profitable deviation for the distinguished player – i.e.* $u_0(a') > u_0(a^*)$ *– if the price never changes* $p(a') = p(a^*)$ *but new ex-post allocation* $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$ *occurs with positive probability. Further, if* $\omega_0^* \neq \alpha_0$*, strategy* $a_0' = 0$ *(not trading) is a beneficial deviation.* □

Proof Since for a stochastic market mechanism ex-post ownership is a random variable we show the first claim of the lemma for any ex-post realization ω_0' with $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$.

For $\omega_0' \neq \omega_0^*$ we can rewrite $\omega_0' = \alpha_0 + x_0^* + \epsilon_x - y_0^* - \epsilon_y$ with $\epsilon_x \neq \epsilon_y$. Then, ex post utility is given as

$$\begin{aligned} u_0(a') &= \omega_0' Ev\left(e(\omega_0')\right) - p^* \cdot (x_0^* + \epsilon_x) + p^* \cdot (y_0^* + \epsilon_y) - c(e\left(\omega_0'\right)) \\ &> \omega_0' Ev\left(e(\omega_0^*)\right) - p^* \cdot (x_0^* + \epsilon_x) + p^* \cdot (y_0^* + \epsilon_y) - c(e\left(\omega_0^*\right)) \\ &= \underbrace{(\omega_0' - \epsilon_x + \epsilon_y)}_{\omega_0^*} Ev\left(e(\omega_0^*)\right) - p^* \cdot (x_0^* - y_0^*) - c(e\left(\omega_0^*\right)) \\ &= u_0(a^*). \end{aligned}$$

for any $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$. The strict inequality $>$ follows since $e(\omega_0^*) \neq e(\omega_0') = \operatorname{argmax}_e \omega_0' E(v\left(e\right)) - c(e)$ and the subsequent equations make use of $p^* = Ev\left(e(\omega_0^*)\right)$ in a true value equilibrium. This implies that the distinguished player improves if $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$ and is unaffected if $\omega_0' \equiv \omega_0^\mu(a') = \omega_0^*$.

For the special case $\omega_0^* \neq \alpha_0$, strategy $a_0' = 0$ (not trading) is a beneficial deviation because then

$$\begin{aligned} u_0(a') &= \alpha_0 Ev\left(e(\alpha_0)\right) - c(e(\alpha_0)) \\ &> \alpha_0 Ev\left(e(\omega_0^*)\right) - c(e\left(\omega_0^*\right)) \\ &= \left[\omega_0^* - x_0^* + y_0^*\right] Ev\left(\omega_0^*\right) - c(e\left(\omega_0^*\right)) \\ &= \omega_0^* Ev\left(\omega_0^*\right) - p^* \cdot (x_0^* - y_0^*) - c(e\left(\omega_0^*\right)) \\ &= u_0(a^*) \end{aligned}$$

for the same reasons as in the first part of the lemma. ■

Now we proceed by considering the following 3 cases. (i) ω_0 is deterministic and the distinguished player always trades, (ii) ω_0 is deterministic and the distinguished player never trades, or (iii) ω_0 is stochastic.

(i) Suppose first ω_0 is deterministic and $\omega_0^* \neq \alpha_0$. From lemma 1 it follows that $a_0' = 0$ is a beneficial deviation and hence a^* cannot be a true value equilibrium.

(ii) Now, suppose ex post ownership ω_0^* is deterministic and $\omega_0^\mu(a^*) = \alpha_0$. Consider first the case where $d(p^*) = 0$ where nobody submits limit buy orders. Since we are looking at a trade equilibrium, there must exist fill-or-kill orders that are executed. In this case the distinguished player can mimic one fill-or-kill buy order that is executed with positive probability. By definition of the market mechanisms this will not have a price impact and the deviating fill-or-kill order of the distinguished player will be executed with positive probability. This would constitute a beneficial deviation due to lemma 1. Therefore, in a true value equilibrium holds $d(p^*) > 0$ if ω_0^* is deterministic.

We next show non-negative excess limit-order-demand $d^*(p^*) \geq s^*(p^*)$. If to the contrary $d^*(p^*) < s^*(p^*)$ the distinguished player can improve buy submitting an order $a_0' = \beta_0' = (s^*(p^*) - d^*(p^*), p^*)$. As a result, his order will be served and the price will not change. Again, by virtue of lemma 1, a_0' is a beneficial deviation and hence a^* not an equilibrium.

Next we claim that if $d^*(p^*) \geq s^*(p^*)$ there must exist a buy order $\beta_i^* = (x, p^*)$ with price limit p^* at the equilibrium price for some positive quantity $x > 0$ which is partially or fully executed with positive probability. Suppose not. Then, $d(p^* + \delta) \geq s(p^*)$. Since $s(p^*) \leq s(p^* + \delta)$ it follows that $\tau(p^*) \leq \tau(p^* + \delta)$. Clearly, $\tau(p^*) < \tau(p^* + \delta)$ is a contradiction of rule 1 from the market mechanism to maximize trade volume. Hence, $\tau(p^*) = \tau(p^* + \delta)$. But then, p^* is picked because the surplus at p^* (the number of unexecuted orders given, price setting rule 2) is (weakly) smaller at p^* than at $p^* + \delta$. The surplus at $p^* + \delta$ is given as $s(p^* + \delta) - d(p^* + \delta)$. This implies that an equilibrium seller can submit a deviating order $a_i' \cup \beta_i'$ with $\beta_i' = (x, p^*)$ where $x > s(p^* + \delta) - d(p^* + \delta)$. This order results in a price increase from p^* to $p^* + \delta$ because now the surplus is greater at p^*. This leads to an improvement for the seller since he can now sell at a higher price (his deviating buy order will not be executed and his equilibrium sell order will be executed due to price priority). This shows that there exists a buy order $\beta_i^* = (x, p^*)$ for some $x > 0$ which is partially or fully executed with positive probability.

This implies that $\tau^*(p^* + \delta) < \tau^*(p^*)$. Otherwise, no buy order using p^* as a price limit will be executed: All buy orders using $p^* + \delta$ as price limit are executed due to price priority.

Now, we claim that the distinguished player can construct a deviating buy order $a_0' = a_0^* \cup \beta_0'$ with $\beta_0' = (x, p^*)$ where x is chosen by some other outside investor who is served in the candidate equilibrium with positive probability. As a result, the price will not change (note that $\tau(p^* + \delta | a') < \tau^*(p^* | a')$ continues to hold since the LHS of this equation is not affected by the additional order). Since both orders have the same priority, the distinguished player will now be served with positive probability. Again, lemma 1 implies that the distinguished player improves. Hence, it cannot be an equilibrium that the distinguished player never trades.

(iii) Next, suppose ex post ownership ω_0^* is stochastic. We argue that this cannot be an equilibrium in a sequence of steps. First, we note that there must exist some outside investors whose ex post ownership structure is also stochastic. Then, we show that the ex post ownership of the distinguished player must be a two point distribution (i.e. either he buys a block or he sells a block). Finally, if the distinguished player buys shares, he must be the only buyer (among the set of players with stochastic ownership) who buys and if he sells he must be the only seller (among the set of investors with stochastic ex post ownership). This implies that all agents with stochastic ownership can improve: Buyers only buy if the distinguished player does not buy, hence they buy only overvalued shares. For sellers the same argument apply and they only do not sell if shares are overvalued and they are better off always selling which can be implemented by a deviating strategy.

It is helpful to start with the following lemma.

Lemma 2 *Consider any candidate true value equilibrium a^* with stochastic $\omega_0^* \in [\omega_0^{min}, \omega_0^{max}]$ where $\omega_0^{min} < \omega_0^{max}$.*

(a) Then, for any $\omega_0' \in (\omega_0^{min}, \omega_0^{max})$ with $\Pr(\omega_0^* = \omega_0') > 0$ it is true that $u_0(a^* | \omega_0^* = \omega_0') < \max\{u_0(a^* | \omega_0^* = \omega_0^{max}), u_0(a^* | \omega_0^* = \omega_0^{min})\}$.

(b) The distinguished player can find strategies a_0' for which $\lim_{\delta \to 0} u_0(a_0') \geq u_0(a^* | \omega_0 = \omega_0^{min})$ if $\omega_0^{min} \leq \alpha_0$ and $\lim_{\delta \to 0} u_0(a_0') \geq u_0(a^* | \omega_0 = \omega_0^{min})$ if $\omega_0^{max} \geq \alpha_0$.

(c) For small enough tick size δ, ex post ownership of the distinguished player follows a two point distribution, i.e. $\omega_0^* \in \{\omega_0^{min}, \omega_0^{max}\}$ with $\omega_0^{min} \leq \alpha_0 \leq \omega_0^{max}$. Furthermore, the distinguished player is indifferent between ω_0^{min} and ω_0^{max}: $u_0(a^* | \omega_0^* = \omega_0^{max}) = u_0(a^* | \omega_0^* = \omega_0^{min})$.

(d) It is true that $\omega_j^* = \omega_j^{max} \Rightarrow \omega_0^* = \omega_0^{min}$ *for all outside investors* $j \neq 0$ *who have stochastic ex post ownership.*

Proof (a) Suppose first that $p^* \leq E(v(\omega_0'))$, i.e. shares are undervalued if $\omega_0 = \omega_0'$. Then, $u_0(a^*|\omega_0^* = \omega_0') \leq \omega_0^{max} \cdot E(v(e(\omega_0'))) - p^*[\omega_0^{max} - \alpha_0] - c(e(\omega_0')) < u_0(a^*|\omega_0^* = \omega_0^{max})$ because $e(\omega_0') \neq e(\omega_0^{max})$. If $p^* \geq E(v(\omega_0'))$ we get $u_0(a^*|\omega_0^* = \omega_0') \leq \omega_0^{min} \cdot E(v(e(\omega_0'))) - p^*[\omega_0^{min} - \alpha_0] - c(e(\omega_0')) < u_0(a^*|\omega_0^* = \omega_0^{min})$ for the same reasons.

(b) Our claim is trivially true if $\omega_0^{min} = \alpha_0$ or $\omega_0^{max} = \alpha_0$. Then, not submitting an order $a_0' = 0$ yields $u_0(a') = u_0(a^*|\omega_0 = \alpha_0)$.

The distinguished player submits a limit order using p^* *as limit price which is randomly executed.* Suppose first an order mimicking $\omega_0^{max} > \alpha_0$. Since a limit order is rationed, it follows that $d(p^*) > s(p^*)$ and $\tau^*(p^*) \geq \tau^*(p^* + \delta)$. (using similar arguments as applied in the proof of part(ii) of this proposition.) The distinguished player can now submit a buy order $\beta_0' = (x, p^* + \delta)$ where x is appropriately chosen to guarantee that $\alpha_0 + x - y^* = \omega_0^{max}$. Note that ex post ownership $\omega_0' = \omega_0^{max}$ and hence $u_0(a') = \omega_0^{max} \cdot E(v(\omega_0^{max})) - p(a')(\alpha_0 - \omega_0^{max})$. Note also that $p(a') \in \{p^*, p^* + \delta\}$ because $\tau(p^* + \delta|a')$ (weakly) increases and $\tau^*(p^*)$ does not decrease. Hence, either $u_0(a') = \omega_0^{max} \cdot E(v(\omega_0^{max})) - p^*(\alpha_0 - \omega_0^{max})$ and the claim holds for any element of the sequence or $u_0(a') = \omega_0^{max} \cdot E(v(\omega_0^{max})) - (p^* + \delta)(\alpha_0 - \omega_0^{max})$ and for the limit of the sequence $\delta \to 0$ the claim is true.

Next, consider mimicking ($\omega_0^{min} < \alpha_0$). Then, the distinguished player can now submit a sell order $\sigma_0' = (y, p^* - \delta)$ with y appropriately chosen to guarantee that $\alpha_0 + x^* - y = \omega_0^{min}$. Similar arguments now imply that the distinguished player now always has $\omega_0 = \omega_0^{min}$ and the price decreases by at most δ.

The distinguished player does not submit a limit order using p^* *as limit price which is randomly executed.* If a limit order is not rationed, the distinguished player only submits fill or kill orders that are executed stochastically. Now, the distinguished player can replace any set of fill or kill orders that are executed if $\omega_0 = \omega_0^{min}$ (resp. $\omega_0 = \omega_0^{max}$) by limit orders without the fill or kill provision that use p^* as the price limit. If the price does not change ($p(a') = p^*$), these limit orders will always be executed because they have higher priority than fill or kill orders and the claim is shown.

If the price changes due to the proposed deviating strategy, a slightly more complicated strategy must be used to guarantee ω_0^{min} or ω_0^{max}. Consider first $\omega_0^{min} < \alpha_0$. Note first that a change in price implies that $\tau^*(p^*) = \tau^*(p^* - \delta)$ due to trade volume maximizing. Furthermore, $|d^*(p^*) - s^*(p^*)| \leq |d^*(p^* - \delta) - s^*(p^* - \delta)|$. Now, consider any set of fill or kill orders submitted by the distinguished player that leads to ω_0^{min}. This set of fill or kill orders is now replaced by limit orders without the fill or kill provision using p^* as the limit price (for example, $\sigma_0' = (y, p^*)$ with y chosen appropriately to guarantee ω_0^{min}). Clearly, $\tau(p^*|a') \geq \tau^*(p^*)$. This simple strategy is complemented by a buy order $\beta_0'' = (x'', p^* - \delta)$ with x'' sufficiently large. Since, $\tau(p^*|a') \geq \tau^*(p^*) \geq \tau^*(p^*)$, sufficiently large x'' now guarantees that the surplus is also smaller at p^* under the deviating strategy a'. Then, the price does not change and indeed $u_0(a') = u_0(a * |\omega_0^{min})$.

Next, consider mimicking $\omega_0^{max} > \alpha_0$. Note first if $d^*(p^*) < s^*(p^*)$, the distinguished player can pick any set of executed fill or kill orders executed that leads to ω_0^{max} and replace them by limit orders with the same quantities that use p^* as the limit price. Since this will increase demand $d(p^*|a') > d^*(p^*)$ a price change does not follow. Hence, we are now concerned with the case that $d^*(p^*) \geq s^*(p^*)$. Suppose first that $d^*(p^*) > s^*(p^*)$. Then, the distinguished player can submit buy orders using $p^* + \delta$ as price limit. As a result, the price will increase by at most δ and $\omega_0 \geq \omega_0^{max}$ (the inequality may occur if demand of fill-or-kill orders is reduced at $p^* + \delta$ and the distinguished player has to submit large enough buy orders to guarantee that fill or kill sell orders can be executed.) If $\omega_0(a') > \omega_0^{max}$ note that $p^* < E(v(e(\omega_0^{max})))$ and the distinguished player can buy even more undervalued shares.

Finally, suppose that $d^*(p^*) = s^*(p^*)$ (this can in particular occur if $\tau^*(p^*) = 0$). Since $s^*(p^*) \leq s^*(p^* + \delta)$ it follows that an increase in the share price can only occur due to an increase of demand at p^* if $d^*(p^*) = d^*(p^* + \delta)$. In that case, however, it must be the case that the surplus at $|s^*(p^* + \delta) - d^*(p^* + \delta)| > 0 = |d^*(p^*) - s^*(p^*)|$ due to the last price setting rule since $p^* < \bar{v}$ for a true value equilibrium if ex post ownership $\omega_0 < 1$ with positive probability. From this, it follows $s^*(p^* + \delta) > d^*(p^* + \delta)$ and $\tau(p^* + \delta | a') > \tau(\tilde{p})$ for all $\tilde{p} \notin \{p^*, p^* + \delta\}$. Hence, a buy order using $p^* + \delta$ as a price limit leads to a price increase of at most δ and it will be served since limit orders have higher priority than fill or kill orders.

(c) If the claim is not true, combining (a) and (b) implies that the distinguished player can always find a deviating strategy by mimicking ω_0^{min} or ω_0^{max}. Also, if the distinguished player is not indifferent between ω_0^{min} and ω_0^{max} he can pick a mimicking strategy that approximate the ex post ownership that leads to a higher utility. Note that the distinguished player cannot be indifferent between ω_0^{min} and ω_0^{max} if either $\omega_0^{min} > \alpha_0$ or $\alpha_0 > \omega_0^{max}$.

(d) Suppose that the claim $\omega_j^* = \omega_j^{max} \Rightarrow \omega_0^* = \omega_0^{min}$ for all $j \neq 0$ does not hold. Then, there exist a $k \neq 0$ for which $\omega_k^* = \omega_k^{max}$ *and* $\omega_0^* = \omega_0^{max}$. This implies that outside investor k submits a buy order which is served while $\omega_0^* = \omega_0^{max}$ or outside investor k submits a sell order which is not served while $\omega_0^* = \omega_0^{max}$. Then, the distinguished player can choose the deviating strategy a_0' which mimicks ω_0^{max} as described under (b) and complement this with a buy order $\beta_0'' = (x'', p')$ where p' is the price used to guarantee ω_0^{max} as derived in (b) and $x'' = \omega_k^{max} - \omega_k^{min}$. This increases utility from the distinguished player because he can now buy more undervalued shares.

This completes the proof of the lemma. ■

From lemma 2 it follows that outside investors who submit a buy order which leads to stochastic ex post ownership can benefit from not buying. Their buy order is only executed if $\omega_0^* = \omega_0^{min}$. However, in a true value equilibrium $p^* = \Pr(\omega_0^* = \omega_0^{min}) \cdot E(v(e(\omega_0^{min}))) + \Pr(\omega_0^* = \omega_0^{max}) \cdot E(v(e(\omega_0^{max})))$. Since $E(v(e(\omega_0^{max}))) > E(v(e(\omega_0^{min})))$ this implies $p^* > E(v(e(\omega_0^{min})))$ and buying outside investors only buy overvalued shares. Hence, not buying constitutes an improvement.

Outside investors who sell stochastically can improve by employing the following deviating strategies. If the distinguished player submits fill or kill orders that are executed randomly, selling outside investors can deviate from submitting fill or kill orders themselves and submit a limit sell order using p^* as the price limit. This limit sell order is accompanied by a large enough (unexecuted) buy order using $p^* - \delta$ as price limit which then guarantees that the price does not decrease. Then, the limit sell order will always be executed before the distinguished player sells his shares. If the distinguished player submits limit orders, outside investors can reduce the price limit of their limit sell orders to $p^* - \delta$. As a result, their order will be executed due to price priority and the price decrease is at most δ. This completes the proof that trade at the true value is not an equilibrium if ex post ownership is stochastic.

Ex post ownership of the distinguished player can neither be deterministic nor stochastic in a true value equilibrium which proves that trade at the true value is not supported by equilibrium behavior.

(II) To prove existence of the excess returns equilibrium we construct equilibrium strategies as follows. The distinguished player submits a buy order for one share $a_0^* = \beta_0^* = (1, \hat{p})$ for any $\hat{p} \in]Ev(e(\alpha_0)), \bar{p}[$ with $\bar{p}$ sufficiently close to $Ev(e(\alpha_0))$ and for small enough δ there exist such $\hat{p}$. It turns out that $\bar{p} \in]Ev(e(\alpha_0)), Ev(e(\alpha_0 + 1/M))[$. Furthermore, one outside investor submits a sell order for one share using $\hat{p}$ as the price limit $a_i^* = \sigma_i^* = (1, \hat{p})$.

Every market mechanism sets $p^* = \hat{p}$ since all other prices lead to zero trade volume and we are looking at an excess returns equilibrium because $\omega_0^* = \alpha_0 + 1/M$, hence $p^* < E(v^*)$ and $\omega \neq \alpha$. Outside investors cannot benefit from increasing demand as this is only feasible if they buy instead of the distinguished player. Then, shares are worth $Ev(e(\alpha_0)) < \hat{p}$ which is thus not a profitable deviation. The equilibrium seller cannot benefit from not selling since then the share not sold will be worth $Ev(e(\alpha_0)) < \hat{p}$.

What remains to be shown is that the distinguished player cannot benefit from not trading. This is true if

$$\begin{aligned} & \alpha_0 \cdot E(v(e(\alpha_0))) - c(e(\alpha_0)) \\ < {} & (\alpha_0 + 1/M) \cdot E(v(e(\alpha_0 + 1/M))) - p^* \cdot (1/M) - c(e(\alpha_0 + 1/M)) \end{aligned}$$

This inequality holds for $p^* = E(v(e(\alpha_0)))$ because $e(\alpha_0) \neq e(\alpha_0 + 1/M)$ and it therefore also holds for a p^* sufficiently close to $E(v(e(\alpha_0)))$. ■

PROOF (OF THEOREM 2, PAGE 17) 1. Suppose conversely that the distinguished player $i = 0$ would exert low effort $e^* = 0$. Then, any equilibrium seller can improve by not selling.

2. For the distinguished player $i = 0$ to exert effort $e^* > 0$ and to be a net seller $\omega_0^{\mu}(a^*) < \alpha_0$ is not optimal, since by not selling and exerting effort $e^* > 0$ he can improve. However, we have seen in 1. that $i = 0$ exerts effort $e^* > 0$. This proves 2. and shows that there must be other players being equilibrium sellers. Otherwise the trade volume would be 0.
3. Suppose not. Then, any non-pivotal equilibrium seller could benefit from not selling shares since they are worth more than the equilibrium price. All non-pivotal investors could benefit from increasing demand if they could buy at the equilibrium price.
4. Sellers only sell in equilibrium if they are pivotal with respect to the distinguished player's $i = 0$ effort choice. Since the market mechanism satisfies voluntary trade, this can only happen if the distinguished player submits an order.
5. An equilibrium winner satisfies $[\bar{v} - p^* - \gamma] \, x_i^* - [\bar{v} - p^*] \, y_i^* > 0$ and violates traditional no-arbitrage by definition:

$$\begin{aligned} [\bar{v} - p^* - \gamma] \, x_i^* - [\bar{v} - p^*] \, y_i^* &> 0 \Leftrightarrow \\ [p^{\mu}(a^*) - v(a^*)] \, y_i^{\mu}(a^*) + [v(a^*) - p^{\mu}(a^*) - \gamma] \, x_i^{\mu}(a^*) &> 0. \end{aligned}$$

■

PROOF (OF PROPOSITION 1, PAGE 17) Let $\Delta v = 0$. This means that $e^* = 0$ which is a contradiction for a^* to be an excess returns equilibrium due to Theorem 2 part 1. ■

PROOF (OF THEOREM 3, PAGE 19) Structure of the proof. We proceed by defining an economy with ownership structure and budget constraints. For this economy, we propose a candidate equilibrium strategy profile a^* for the distinguished player and rational investors. The task is to specify everything appropriately such that one finds a strategy profile a^* that establishes an excess returns equilibrium. The proof contains two auxiliary results lemmata 3 and 4. Lemma 3 shows that there exist exogenous parameters such that the candidate equilibrium yields strictly positive excess returns and the critical non-deviation condition can be satisfied at the same time. Finally, lemma 4 shows that for candidate equilibrium strategy profile a^* there are no strictly improving deviations and thereby that indeed it forms a Nash equilibrium given the parameters satisfy the conditions of lemma 3. The proofs in part (i) for μ_N and parts (ii) and (iii) for μ_S and μ_T only differ in the specification of the distinguished player's equilibrium strategy and in the budget constraints. The equilibrium strategy is constructed such that the distinguished player sells either his entire stake or nothing. Thereby the distinguished player's optimal effort and the company value can assume just two states of nature. Accordingly, we call $e^h = e(\alpha_0)$ *high effort* and $e^l = e(0) = 0$ *low effort*. High effort is given by

$$e^h = \arg\max_e \alpha_0(\underline{v} + e(\bar{v} - \underline{v})) - ce^2 = \frac{\alpha_0(\bar{v} - \underline{v})}{2c}.$$

Define $\hat{v} := \underline{v} + e^h(\bar{v} - \underline{v}) = \underline{v} + \frac{\alpha_0(\bar{v}-\underline{v})^2}{2c}$ as the company value if the distinguished player exerts high effort. The distinguished player is indifferent between e^h and e^l at price $p = \tilde{p}$ for

$$\alpha_0\hat{v} - c(e^h)^2 = \alpha_0\tilde{p}.$$

This yields a critical price

$$\tilde{p} = \underline{v} + \frac{\alpha_0(\bar{v} - \underline{v})^2}{4c} = \hat{v} - \frac{\alpha_0(\bar{v} - \underline{v})^2}{4c} = \frac{1}{2}(\underline{v} + \hat{v}) \tag{1}$$

above which the distinguished player prefers to sell his stake and then to exert low effort $e^l = 0$. Note that $\tilde{p}$ may not be an element in P. Accordingly denote by

$$p_0 := \min_{p\in P} \{p \,| p > \tilde{p}\}$$

the lowest discrete price at which the distinguished player has a strict incentive to sell instead of exerting high effort, i.e. $0 < p_0 - \tilde{p} \leq \delta$.

(i)
1. **The economy.** Small rational outside investors[26] are specified with initial stake $\alpha_i \; \forall i \in (0,1)$ with aggregated stake $\int_{(0,1)} \alpha_i di = \alpha_r$ and budget constraint B_i with aggregated constraint $\int_{(0,1)} B_i di = B > \bar{v}$. This specification implies that the aggregated budget constraint is never binding since together small investors can afford to buy strictly more than the entire firm at the highest value. However for any single investor there is a finite upper bound B_i up to which buy orders can be submitted.[27]

2. **Equilibrium strategies.**

   DP Distinguished player
   $a_0^* = \sigma_0$ with $\sigma_0\,(p_0) = (\alpha_0, p_0)$.

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[26] When we talk about the stakes of small continuum investors we mean infinitesimal stakes. For example, a measurable subset J of small investors jointly owns $\int_J \alpha_i di = \int_J 1 d\alpha$ stakes. For example, if J has Lebesgue measure $\lambda(J)$ and every investor owns the same α_r infinitesimal stakes then they jointly own $\lambda(J) \cdot \alpha_r$ stakes. The same holds for infinitesimal budgets.

[27] The following interpretation of the budget constraint matters for the proof. Though B_i can be arbitrarily large, we suppose that the market mechanism does not execute buy orders $\beta_i = (b, p)$ with $b \cdot p > B_i$ of small investors since they could not not afford them unless they submit sell orders that are executed. Since traders don't know in advance if and for which price their sell order is executed – which could be random – buy orders are not allowed to be based on them. This assumption excludes equilibrium deviations where traders behave as bellers – i.e. buyers and sellers at the same time in order to increase their budget.

The distinguished player $i = 0$ submits a single order using p_0 as the limit price to sell his entire shares. The price limit of the distinguished player is p_0, i.e. the lowest discrete price at which he has a strict incentive to sell instead of exerting high effort.

ROI Rational outside investors
$a_i^* = \{\beta_i^*, \sigma_i^*\} = \{(\rho_i, p_0 - \delta), (\alpha_i, p_0)\}$ for $i \neq 0$ with $\rho_i := \frac{B_i}{p_0 - \delta}$.
Here, ρ_i is the maximal buy order at price $p_0 - \delta$ any rational buyer can afford according to his budget constraint. Outside investors submit maximal buy orders for ρ_i shares using $p_0 - \delta$ as a price limit and sell orders for all their shares using p_0 as the limit price.

3. **Allocation and prices**

The market mechanism together with the action profile specifies the expected equilibrium price, the value of the firm – implicitly determined by the ex post stakes owned by the distinguished player – and which outside investors' orders are executed determined by stochastic rationing.

States of nature.

For this strategy profile three relevant realizations of noise can be relevant.

(1) noise traders sell in aggregate with probability $\theta_1 := \Pr(\tilde{Z}_1 < 0)$. If noise traders buy in aggregate the outcome depends further on the probability $\Pr(\omega_0 \neq \alpha_0)$ that the distinguished player can sell.

(2) Denote by $\theta_2 := \Pr(\tilde{Z}_1 \geq 0 \texttt{ and } \omega_0 = \alpha_0)$ the probability that noise traders buy and the distinguished player does not sell and respectively by

(3) $\theta_3 := 1 - \theta_1 - \theta_2 = \Pr(\omega_0 \neq \alpha_0)$ the probability that the distinguished player sells — which can only occur if noise traders buy sufficiently much. The probability that the distinguished player cannot sell is then given by $1 - \theta_3 = \theta_1 + \theta_2$.

Expected price $E(p)$.

In this strategy profile rational investors and the distinguished player never trade with each other. Therefore, market mechanism $\mu = \mu_N$ picks $p^\mu = p_0 - \delta$ if $\tilde{Z}_1 < 0$ and $p^\mu = p_0$ if $\tilde{Z}_1 > 0$. This yields an expected price

$$E(p) = \theta_1 \cdot (p_0 - \delta) + (\theta_2 + \theta_3) \cdot p_0.$$

Expected company value $E(v)$.

The expected value of the company depends only on the distinguished player's final stake ω_0. Since the market mechanism guarantees fully executed orders, the distinguished player either sells all his shares or none at all. Consequently, the distinguished player exerts effort e^h if $\omega_0 > 0$. This implies an expected value

$$E(v) = (\theta_1 + \theta_2) \cdot \hat{v} + \theta_3 \cdot \underline{v}.$$

It is important to note that $\theta_3 > 0$ as long as $\alpha_0 < b$, i.e. α_0 is small enough such that the distinguished player can sell all his shares to noise traders.

Excess returns.

Excess returns are defined as $R(p) = E(v) - E(p)$ where

$$\begin{aligned} R(p) : &= \theta_1 \cdot (\hat{v} - p + \delta) + \theta_2 \cdot (\hat{v} - p) + \theta_3 \cdot (\underline{v} - p) \qquad (2) \\ &= \theta_1 \cdot (\hat{v} + \delta) + \theta_2 \cdot \hat{v} + \theta_3 \cdot \underline{v} - p. \end{aligned}$$

Rationing.

To determine the rationing parameters consider again the 3 relevant events:

(1) With probability θ_1 the market mechanism realizes price $p_0 - \delta$ and aggregated demand is $\rho = \int_{(0,1)} \rho_i$ while expected aggregated supply is $Z_- := E\left(\|\tilde{Z}_1\| \,\middle|\, \tilde{Z}_1 < 0\right)$. This implies that every rational buyer will be served with probability

$$\lambda_1 := \frac{Z_-}{\rho}.$$

Since $\rho > 1$ and $0 < Z_- < 1$ there will always be rationing, i.e. $0 < \lambda_1 < 1$ in this event.

(2) With probability θ_2 price p_0 is realized and expected aggregated demand is $Z_+ := E\left(\tilde{Z}_1 \,\middle|\, \tilde{Z}_1 \geq 0\right)$ while aggregated supply is α_r. Every small outside investor is then served with probability

$$\lambda_2 := \frac{Z_+}{\alpha_r}.$$

(3) With probability θ_3 price p_0 is realized and expected aggregated demand is $Z_{++} := E\left(\tilde{Z}_1 \,\middle|\, \tilde{Z}_1 \geq 0 \texttt{ and } \omega_0 \neq \alpha_0\right)$. Since the distinguished player now can sell his shares remaining demand has to be reduced by α_0. Again, aggregated supply is α_r from rational investors. Every small outside investor is then served with probability

$$\lambda_3 := \frac{Z_{++} - \alpha_0}{\alpha_r}.$$

Lemma 3 *For any non-degenerate symmetric noise and small enough δ, there exists α_0, and c such that conditions*

$$\begin{aligned} R(p_0) &> 0, & (3)\\ \theta_3\lambda_3 &\geq \theta_2\lambda_2 & (4)\\ \theta_1 \cdot \lambda_1 \cdot (\hat{v} - p_0 + \delta) &\geq R(p_0), & (5) \end{aligned}$$

are satisfied. □

Condition (3) guarantees strictly positive excess returns while inequalities (4) and (5) make sure that the critical non-deviation conditions for rational outside investors in lemma 4 hold.

Proof (of lemma 3) First of all note that it is possible to choose an $\alpha_0 > 0$ to ensure

$$\theta_3 > \max\left\{\frac{\lambda_2\theta_2}{\lambda_3}, \frac{1-\theta_1\lambda_1}{2}\right\} \texttt{ and } \theta_3 < \frac{1}{2} \tag{6}$$

This is achieved by choosing an α_0 which guarantees that θ_3 is close to 1/2. This is possible since for symmetric and continuous non-degenerate noise the likelihood that the distinguished player can sell can be increased arbitrarily close to $\frac{1}{2}$ if his stake α_0 gets small enough since $\theta_3 = \Pr(\tilde{Z}_1 \geq 0 \texttt{ and } \omega_0 \neq \alpha_0) \leq \Pr(\tilde{Z}_1 \geq 0) = 1/2$. As for the first inequality, note that as $\theta_3 \to 1/2$ it must be that $\theta_2 \to 0$ due to the fact that $\theta_2 + \theta_3 = 1/2$. The second inequality follows since $\frac{1-\theta_1\lambda_1}{2}$ is bounded away from 1/2 and we can choose θ_3 close enough to 1/2 to guarantee that $\theta_3 > \frac{1-\theta_1\lambda_1}{2}$. The last inequality follows by construction since we are choosing a θ_3 close to 1/2 (and $\theta_3 \leq 1/2$).

Given (1) it follows that $0 < R(\tilde{p}) - R(p_0) \leq \delta$ since $0 < p_0 - \tilde{p} \leq \delta$. Equation (2) together with (1) implies

$$\begin{aligned} R(\tilde{p}) &= \theta_1 \cdot (\hat{v} - \tilde{p} + \delta) + \theta_2 \cdot (\hat{v} - \tilde{p}) + \theta_3 \cdot (\underline{v} - \tilde{p}) \\ &= (\theta_1 + \theta_2) \cdot (\hat{v} - \tilde{p}) - \theta_3 \cdot (\tilde{p} - \underline{v}) + \delta\theta_1 \\ &= (1 - 2\theta_3) \cdot \left(\frac{\alpha_0 (\bar{v} - \underline{v})^2}{4c} \right) + \delta\theta_1 \end{aligned} \tag{7}$$

Since $\theta_3 < \frac{1}{2}$ by virtue of (6) and $\frac{\alpha_0(\bar{v}-\underline{v})^2}{4c} > 0$ excess returns are strictly positive for small enough $\delta > 0$, hence condition (3) holds.

Finally (6) says

$$\begin{aligned} \theta_3 - \frac{1 - \theta_1\lambda_1}{2} &> 0 \\ \Leftrightarrow (\theta_1\lambda_1 - 1 + 2\theta_3) \left(\frac{\alpha_0(\bar{v} - \underline{v})^2}{4c} \right) &> 0. \end{aligned}$$

Hence, for small enough δ

$$\begin{aligned} (\theta_1\lambda_1 - 1 + 2\theta_3) \left(\frac{\alpha_0(\bar{v} - \underline{v})^2}{4c} \right) - \delta \cdot \theta_1 \geq 0 & \\ \Leftrightarrow (\theta_1\lambda_1) \left(\frac{\alpha_0(\bar{v} - \underline{v})^2}{4c} \right) &\geq R(\tilde{p}) > R(p_0) \Rightarrow \\ \theta_1\lambda_1 \cdot (\hat{v} - p_0 + \delta) &\geq R(p_0) \end{aligned}$$

which is inequality (5) and in the intermediary step the definition for $R(\tilde{p})$ from equation 7 is used. ■

Lemma 4 *Strategy* $a^* = \{a_0^*, \{a_i^*\}_{i \in (0,1)}\}$ *with* $a_0^* = \sigma_0(p_0) = (\alpha_0, p_0)$ *and* $a_i^* = \{\beta_i^*, \sigma_i^*\} = \{(\rho_i, p_0 - \delta), (\alpha_i, p_0)\}$ *for* $i \neq 0$ *is an equilibrium strategy if constraints (3), (4) and (5) are met and* $\mu = \mu_N$. □

Proof (of lemma 4) We discuss deviations at any equilibrium price that can occur with positive probability.

1. Equilibrium utility of rational outside investor.

   The equilibrium utility of a risk neutral rational outside investor $i \in (0,1)$ is

   $$\begin{aligned} u_i(a^*) = \ & \theta_1 \cdot [(\alpha_i + \lambda_1\rho_i)\,\hat{v} - \lambda_1\rho_i(p_0 - \delta)] \\ & + \theta_2 \cdot \alpha_i \cdot [(1 - \lambda_2) \cdot \hat{v} + \lambda_2 \cdot p_0] \\ & + \theta_3 \cdot \alpha_i \cdot [(1 - \lambda_3) \cdot \underline{v} + \lambda_3 \cdot p_0]. \end{aligned}$$

   In the first expression $\lambda_1\rho_i(\hat{v} - p_0 + \delta)$ are the expected benefits from buying shares at the lower price $p_0 - \delta$ if $\tilde{Z}_1 < 0$. The second and third expressions are the gains from selling shares which can happen if $\tilde{Z}_1 \geq 0$. The second expression is the gain from selling if the distinguished player exerts high effort. If the distinguished player sells his shares he will exert low effort in turn. Again, there is rationing and only a fraction λ_3 of all sell orders can be served in that case.

2. Price $p \geq p_0$.

(a) Outside investor $i \in (0,1)$: No outside investor has an impact on equilibrium price and we can restrict our analysis of deviations to the two realizations of equilibrium prices. Outside investors could increase their demand at $p \geq p_0$ by submitting a buy order $\beta_i' = (b, p_0 + \epsilon)$ with $b > 0$ and $\epsilon \geq 0$.
As a consequence, the buy order would not be rationed due to price priority which is beneficial for the low price and adverse if the distinguished player sells his shares. It is not a beneficial deviation if

$$\theta_1 \cdot \lambda_1 \cdot (\hat{v} - p_0 + \delta) \geq \theta_1 \cdot (\hat{v} - p_0 + \delta) + \theta_2 \cdot (\hat{v} - p_0) + \theta_3 \cdot (\underline{v} - p_0)$$

which holds by inequality (5).
Decreasing sell orders is not beneficial if

$$\begin{aligned} u_i(a^*) - u_i(a') &\geq 0 \Leftrightarrow \\ \alpha_i \left[\theta_2 \left((1-\lambda_2)\hat{v} + \lambda_2 p_0 - \hat{v}\right) + \theta_3 \left((1-\lambda_3)\hat{v} + \lambda_3 p_0 - \underline{v}\right)\right] &\geq 0 \Leftrightarrow \\ \alpha_i \left[\theta_3 \lambda_3 (p_0 - \underline{v}) - \theta_2(\lambda_2(\hat{v} - p_0))\right] &\geq 0, \end{aligned}$$

where a' stands for the deviation where only outside investor i does not sell. The last inequality is equivalent to inequality (4) by applying (1).

(b) Distinguished Player: Offering to sell at higher prices is not beneficial since these orders will not be executed. This is true because we assume that $b \leq \alpha_\rho$ and hence $\tilde{Z}_1 > \alpha_\rho$ is not possible and orders from rational outside investors can always match the noise. Offering to sell only a fraction is not beneficial. Consider any ex post ownership $\omega_0' \neq \alpha_0$ and $\omega_0' \neq 0$. Then, if $v(e(\omega_0')) \leq p_0$ the distinguished player would be better off selling all shares and saving efforts costs (as compared to owning ω_0'. If $v(e(\omega_0')) > p_0$ the distinguished player would be better off not selling any shares (as compared to owning ω_0').

3. Price $p < p_0$.

   (a) Outside investor $i \in (0,1)$: Clearly, increasing supply or decreasing demand at $p < p_0$ is not a beneficial deviation since shares are undervalued for $p < p_0$.

   (b) Distinguished Player: Selling at $p < p_0$ is not beneficial by construction of p_0. ■

This completes the proof of part (i).

(ii) $\mu = \mu_T$: Specify noise $\Pr(\tilde{Z}_1 \in (0,t) = 0)$ for some $t > 0$ and $\alpha_0 \leq t$. Furthermore, budget constraints are as in (i). Finally, the distinguished player now submits the same limit order as in (i). With these re-specifications the proof is the same as in (i).

(iii) Without noise, our call auction market mechanisms obey the voluntary trade property. Hence, we can again apply our characterization result. However, since agents cannot be pivotal in the continuum case, an excess returns equilibrium cannot exist.

(iv) The proof is by contradiction. Suppose to the contrary that there exists an excess returns equilibrium. This implies that shares are traded at a price $p < v = \bar{v} = \underline{v}$. Note first that not every investor can be served with probability one since $\alpha_1 < 1$ but aggregate budget constraint allows rational investors to buy more than the entire firm at prices $p < v$. Therefore, the strictly positive measure of traders who are not served with probability one have an incentive to increase their price limit. This would increase their equilibrium utility since they will now always be served at p by virtue of the price priority property of the market microstructures studied in this section.

This completes the proof of theorem 3. ■

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